Final Exam
MATH 106-D, Fall 2015

Name: ______________________________

Instructions:

- Please answer as many of the following questions as possible.
- No cell phones or collaboration allowed. If you leave the classroom during the exam you must leave your cell phone with the instructor.
- Approved calculators are allowed.
- Additional scrap paper is available upon request.
- Show all of your work on the page of the problem. Clearly indicate your answer and the reasoning that you used to arrive at the answer. You do not have to simplify algebraic expressions.

This exam has 7 problems. There are a total of 100 points.

Good luck!

Problem	**Possible Points**	**Points Earned**
1	18	
2	12	
3	16	
4	12	
5	12	
6	16	
7	14	
TOTAL	**100**	

1. (18 points) Evaluate the following integrals. Show all of your work and do not use Taylor series.

   (a) $\displaystyle\int \frac{e^{2x}}{3(e^{2x}-3)}\,dx$

   (b) $\displaystyle\int (2x+1)\cos(x)\,dx$

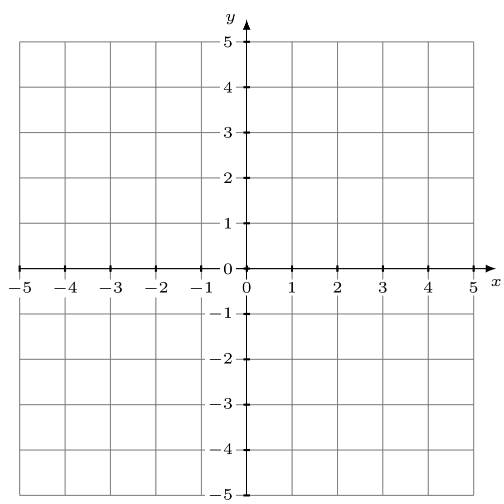
   (c) $\displaystyle\int \frac{dx}{(x^2+1)^{3/2}}$

2. (a) (6 points) Let R_1 be the region in the first quadrant bounded by

$$f(x) = \frac{1}{x}, \quad g(x) = \frac{1}{2x}, \quad x = 1, \quad \text{and} \quad x = 2.$$

Find the volume of the solid obtained by rotating the region R_1 around the x-axis.

(b) (6 points) Let R_2 be the region enclosed by the line $y = x$ and the curve $x = y^2 - 2$. Draw R_2 on the coordinate axis below and label the points of intersection. Then set up **but do not evaluate** an integral that computes the area of R_2. You may choose to integrate with respect to x or y.



3. (a) (6 points) Determine whether the series converges or diverges: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n + 5n}$.

(b) (6 points) Determine whether the series converges or diverges: $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{2k}{2k-1}$.

(c) (4 points) Does the series in part (b) converge absolutely? Explain why or why not.

4. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{(n^2+2)^2}$.

(a) (8 points) Use the Integral Test to determine if the series above converges or diverges. Verify all assumptions of the test.

(b) (4 points) If the series converges, use the Integral Test to find an upper bound for $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{(n^2+2)^4}$. If the series diverges, leave this problem blank.

5. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{3^n}(x-1)^n.$

(a) (6 points) Find the radius of convergence of the power series. Clearly indicate which tests you use.

(b) (6 points) Find the interval of convergence of the power series. Clearly indicate which tests you use, and verify that all of the necessary assumptions are satisfied.

6. Let $f(x) = \dfrac{1}{4x^2 - 1}$ and let N be a natural number ($N = 1, 2, 3, \dots$).

(a) (4 points) Put in increasing order the quantities $I = \displaystyle\int_1^{N+1} f(x)\,dx$, L_N, and R_N.

(b) (4 points) Consider $I = \displaystyle\int_1^{N+1} f(x)\,dx$. The sum $\displaystyle\sum_{n=1}^{N} \frac{1}{4n^2 - 1}$ is equal to one of L_N or R_N. Determine the correct choice and explain your answer.

(c) (6 points) Find a simple expression for $\displaystyle\sum_{n=1}^{N} \frac{1}{4n^2 - 1}$. *Hint: use partial fractions.*

(d) (2 points) Find $\lim_{N\to\infty} \sum_{n=1}^{N} \frac{1}{4n^2-1}$.

Bonus. (4 points) Give an explanation why $\int_1^\infty \frac{1}{4x^2-1}\, dx \leq \lim_{N\to\infty} \sum_{n=1}^{N} \frac{1}{4n^2-1}$.

7. (a) (6 points) What is $\sum_{k=0}^{\infty} 2^{-2k}$?

(b) (6 points) Find the first four non-zero terms of the Maclaurin series for

$$f(x) = x \sin(2x).$$

(c) (2 points) Find $f^{(2016)}(0)$ exactly for $f(x) = x \sin(2x)$.