Math 106AB Name_____
Calculus 2 Mr. Balcomb
Final Exam
December 16, 2015

Show all work for full credit.

1. Let I = $\int_1^2 \frac{1}{x^2} dx$

A. Use the Fundamental Theorem of Calculus to evaluate I exactly.

B. Compute the approximating sum L_4. Show your work.

C. List in increasing order L_4, R_4, T_4 , M_4 .

2. Evaluate the integrals:

   A. $\int \frac{x+2}{x^2-4x}dx$

   B. $\int \cos^3(x)dx$

3. For f(x) = $\ln(x)$,

   a. Give the third degree polynomial for f(x) based at x_0=1.

   b. Use this polynomial to estimate ln(2).

   c. What is the possible error that could have occurred in your estimate in part b? Recall that if you use the Taylor polynomial of degree n at x_0 to approximate f(x) for x in an interval I containing x_0 then

   $\frac{K_{n+1}}{(n+1)!}|x-x_0|^{n+1}$ is an upper bound for the approximate error. [K_{n+1} is an upper bound for the absolute value of the (n+1)st derivative on I.]

4. Find the volume of the solid formed by revolving the region bounded by the graph of $y = \sin(x)$ and $y = 0$ in the interval $[0, \pi]$ about the x-axis.

5. Find the solution that passes through (4,2) for the equation

$$xy' = y$$

6. For the graph f(x) = ln(cosx),

a) write the integral to find the length of the arc from x= 0 to x = $\pi/4$.

b) Evaluate your integral.

7. Do these integrals converge? Evaluate those integrals that do converge. Justify your answer.

a. $\int_0^\infty \frac{1}{x^2+1} dx$

b. $\int_{-1}^{1} \frac{1}{\sqrt[3]{x}}\,dx$

8. For each of the following series, test to see whether it converges absolutely, converges conditionally or diverges and **explain why**.

a. $\sum_{n=0}^{\infty}(-\frac{1}{4})^n$

b. $\sum_{n=1}^{\infty}\frac{2^n+1}{2^{(n+1)}}$

c. $\sum_{j=1}^{\infty} \frac{1}{\sqrt{j+2}}$

9. Consider the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+1)^n}{2^n}$.

a. Give its radius of convergence. Show your work.

b. Give its interval of convergence. Show your work.

10. a) Give the Maclaurin series for sin(x)

b) Find a power series expression for xsin(x).

c) Now find a power series expression for $\int x\sin(x)dx$.

d) Using your formula from part (c), approximate $\int_0^1 x\sin(x)dx$ with an error less than 0.01. Justify your answer.

11. Evaluate $\int \frac{\sqrt{x^2-1}}{x} dx$

12. Does the following integral converge or diverge? Justify your answer.

$$\int_1^\infty \frac{\ln(x)}{x^2} dx$$