

**Math 106: Review for Final Exam, Part II**

1. Use a second-order Taylor polynomial to estimate  $\sqrt[3]{28}$ .

2. What is the largest possible error that could have occurred in your previous estimate?

3. Use a comparison to show whether each of the following converges or diverges. If an integral converges, give a good upper bound for its value.

(a)  $\int_1^{\infty} \frac{7 + 5 \sin x}{x^2} dx$

(b)  $\int_1^{\infty} \frac{1 + 3x^2 + 2x^3}{\sqrt[3]{10x^{12} + 17x^{10}}} dx$

4. Decide if each of the following sequences  $\{a_k\}_{k=1}^{\infty}$  converges or diverges. If a sequence converges, compute its limit.

(a)  $a_k = 3 + \frac{1}{10^k}$

(b)  $a_k = (-1)^k$

(c)  $a_k = \frac{5e^k}{7e^k + \ln(k+1)}$

**Strategy.** The following is a good order in which to consider the various series convergence tests.

- Do the individual terms approach 0? If they don't approach 0, the  $n$ th Term Test tells you the series must diverge. If they do approach 0, try another test.
  - Is the series geometric? (That is, do you multiply by the same constant  $r$  to get from each term to the next?) If so, the series converges if  $|r| < 1$  and diverges otherwise.
  - Does the series contain something such as  $(-1)^k$  or  $(-1)^{k+1}$  or  $\cos(k\pi)$  that makes its terms alternate? If so, try the Alternating Series Test.
  - Does the series contain a factorial ( $k!$ ) or exponential (such as  $2^k$  or  $e^k$ )? If so, try the Ratio Test.
  - If the series has positive terms, does it remind you of a simpler series (especially a  $p$ -series or a geometric series)? If so, try the Comparison Test.
  - Is the formula something you can integrate easily? If so, try the Integral Test.
5. Decide if each of the following series converges or diverges. If a series converges, find its value.

(a)  $3.1 + 3.01 + 3.001 + 3.0001 + \cdots$

(b)  $1 + 1/2 + 1/3 + 1/4 + \cdots$

(c)  $5 - 5/3 + 5/9 - 5/27 + \cdots$

6. Decide if each of the following series converges or diverges. If a series converges, find upper and lower bounds for its value.

(a) 
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[3]{k+1}}$$

(b) 
$$\sum_{k=1}^{\infty} \frac{(2k)!}{3^k (k!)^2}$$

(c) 
$$\sum_{k=1}^{\infty} \left( \frac{1}{100} + \frac{1}{k^5} \right)$$

(d) 
$$\sum_{k=1}^{\infty} \frac{\sqrt{9k^8 + 5k^6}}{12k^5 + 3}$$

(e) 
$$\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$$

7. Does the first series from the previous problem converge absolutely or conditionally?

8. Compute the radius and interval (including endpoints) of convergence for  $\sum_{k=1}^{\infty} \frac{(x+3)^k}{k \cdot 5^k}$ .

9. Evaluate the following exactly.

(a)  $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \cdots$

(b)  $1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \cdots$

10. (a) Using summation notation, write the series equal to  $\int_0^1 e^{-x^2} dx$  and show that it converges.

(b) If  $f(x) = e^{-x^2}$ , what is  $f^{(400)}(0)$ ? What is  $f^{(401)}(0)$ ?