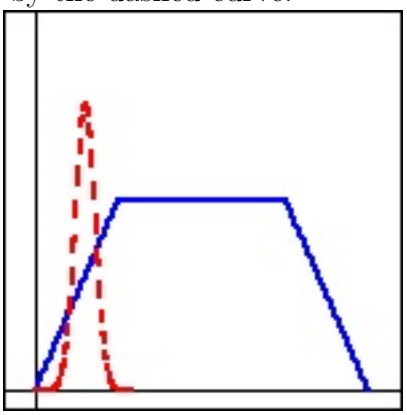
Name:_____

Final Exam

Show all your work to receive full credit for a problem. There are a total of 100 points on this test. Good luck, and thank you for a wonderful semester!

1. (3 points each.) Two cars start at the same time and travel in the same direction along a straight road. The figure below gives the velocity, v (in kilometers per hour), of each car as a function of time (in hours). The velocity of car A is given by the solid curve, and the velocity of car B by the dashed curve.



   (a) Which car attains the larger maximum velocity?

   (b) Which car stops first?

   (c) Which car travels farther?

2. (4 points each.) The table below gives the values of the continuous function h at some x-values.

x	$h(x)$
-5	9.25
-4	8.03
-3	7.05
-2	7.01
-1	1.02
0	0.69
1	1.02
2	5.45
3	-0.76
4	-4.93
5	-2.93

(a) Determine the smallest interval in which the function h *must* have a root. Justify your answer.

(b) Why does this function have to have a minimum and a maximum value? Where do you think h attains its absolute minimum value?

(c) Explain why h must have a stationary point in the interval $(-1, 1)$.

3. (4 points.) Match the functions and their derivatives (you do not need to show work for this problem):

(a) $y = \displaystyle\int_0^x \frac{t^2}{1+t^2}dt$	(I) $y' = \dfrac{2x}{1+x^4}$
(b) $y = \ln(1+x^2)$	(II) $y' = \dfrac{2x}{1+x^2}$
(c) $y = \arctan(x^2)$	(III) $y' = \dfrac{2x}{(1+x^2)^2}$
(d) $y = \dfrac{-1}{1+x^2}$	(IV) $y' = \dfrac{x^2}{1+x^2}$

4. (5 points.) For the equation given below, evaluate $\dfrac{dy}{dx}$ at the point $(1, -2)$.

$$xe^y - 3y = 2x + 4 + e^{-2}$$

5. (5 points.) Find the absolute maximum and absolute minimum values of the function $p(x) = x^3 - 6x^2 - 63x + 3$ over the interval $[-4, 0]$. (Note: You may check your work on your calculator, but to get full credit you must give an algebraic justification for your answer.)

6. (5 points each.) Evaluate the following limits:

(a) $\lim_{x \to 1} \dfrac{e^x - e}{\ln x}$

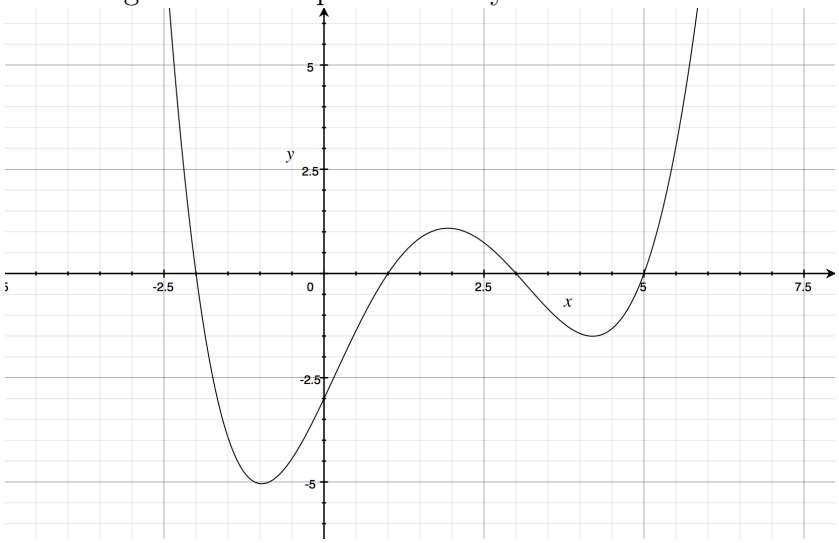
(b) $\lim_{x \to \infty} \dfrac{e^{4x}}{x^2}$

7. (5 points each.) Evaluate the following integrals:

(a) $\displaystyle\int_{-2}^{3} -7t^{-5} dt$

(b) $\displaystyle\int_{1}^{4} 3(2^x) dx$

8. (3 points.) Let f be the function whose graph is shown below, and let $A_f(x) = \int_0^x f(t)dt$. List the following quantities in order from smallest to largest, and use the space provided to give a brief explanation of your answer.



(a) $A_f(1)$

(b) $A_f(3)$

(c) $A_f(5)$

(d) $A_f(6)$

9. (3 points.)Consider the integral $\int_0^{\pi} \sin(5x)dx$. Which of the following correctly expresses the integral as a limit of Riemann sums? Briefly justify your answer.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\pi + \frac{5\pi i}{n}\right)$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{\pi i}{n}\right)$

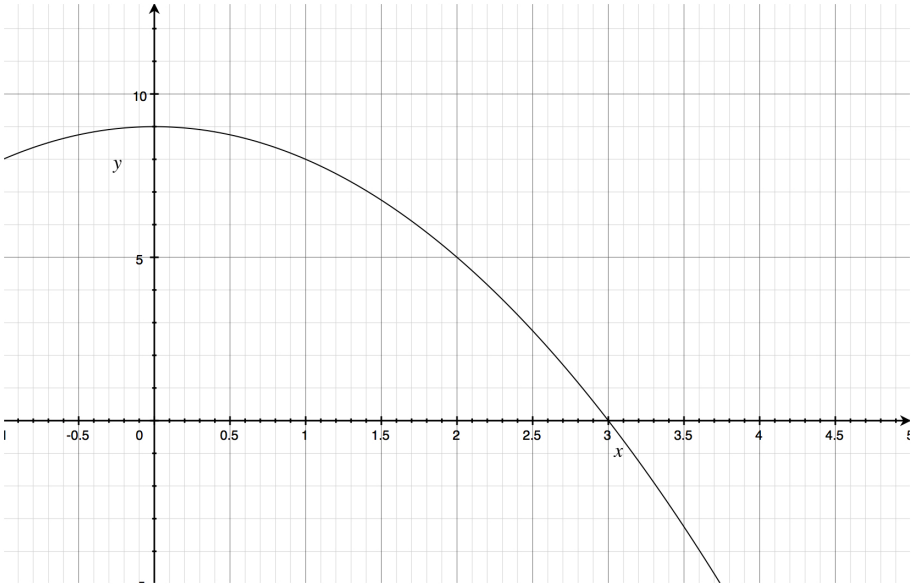
(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\frac{5\pi i}{n}\right)$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{5\pi i}{n}\right)$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\frac{\pi i}{n}\right)$

(f) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\pi + \frac{5\pi i}{n}\right)$

10. (4 points each.) Consider the function $g(x) = 9 - x^2$ on the interval $[0, 3]$. A graph of g is provided below.

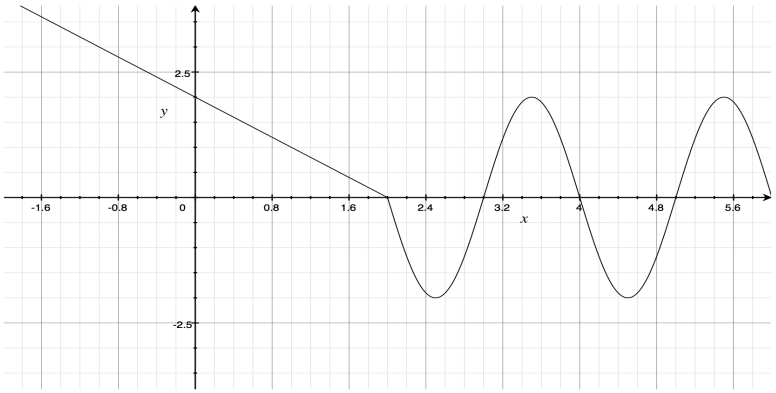


(a) Partition the interval into six equal subintervals and use left approximating sums to compute an estimate of $\int_0^3 g(x)dx$. Make sure to draw the rectangles in the picture above.

(b) Use the Fundamental Theorem of Calculus to calculate the exact value of the integral $\int_0^3 g(x)dx$.

(c) Is the estimate in part (a) an overestimate or an underestimate? Explain why you should have expected this from the graph.

11. (3 pts each) Let $G(x) = \int_0^x g(t)dt$ where $g(t)$ is the function shown in the figure. Answer the following questions about G. (You may use estimates if you're not sure of the exact values.)



(a) What is $G(-1)$? $G(1)$? $G(2)$?

(b) What is $G'(1)$? What is $G''(1)$?

(c) Where are the stationary points of G?

(d) On what intervals is G increasing? On what intervals is G decreasing?

(e) What are the inflection points of G?

(f) On what intervals is G concave up? On what intervals is G concave down?

12. (9 points.) At noon, ship A is 40 nautical miles due west of ship B. Ship A is sailing west at 16 knots and ship B is sailing north at 17 knots. How fast (in knots) is the distance between the ships changing at 4pm? (Note: 1 knot is a speed of 1 nautical mile per hour.)