

Calculus I - Section A Chapters {2,3,4,5} Proficiency Assessment  
December 15, 2015

Name: \_\_\_\_\_

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1. **Do not open this booklet until you are told to do so.**
  2. Try not to separate the pages. If they do become separated, write your names on every page and point this out to your proctor when you hand it in.
  3. Show an appropriate amount of work (including appropriate explanation) for each problem and not just the final answer. Include units in your answer where that is appropriate.
  4. You may use any calculator functionally equivalent to a TI-83/TI-83+ or TI-84/TI-84+. Use of calculators with more functionality than these is not allowed.
  5. **Turn off all cell phones and pagers, and remove all headphones.**
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**Proficiency Level on Chapter 2:**\_\_\_\_\_

**Proficiency Level on Chapter 3:**\_\_\_\_\_

**Proficiency Level on Chapter 4:**\_\_\_\_\_

**Proficiency Level on Chapter 5I:**\_\_\_\_\_

**Proficiency Level on Chapter 5II:**\_\_\_\_\_

**Proficiency Level on Chapter 5:**\_\_\_\_\_

**Chapter 2 Proficiency Test - Problem 1****(III)** Find

$$\lim_{x \rightarrow 2^-} \frac{|x-2|}{x-2}, \quad \lim_{x \rightarrow 2^+} \frac{|x-2|}{x-2}, \quad \text{and} \quad \lim_{x \rightarrow 2} \frac{|x-2|}{x-2}.$$

**(II)** It can be shown that the inequalities

$$\frac{1}{2} - \frac{(x-2)^2}{6} < \frac{(x-2) \sin(x-2)}{2 - 2 \cos^2(x-2)} < \frac{x^2}{4} - x + \frac{3}{2}$$

hold for all values of  $x$  close to two. What, if anything, does this tell us about

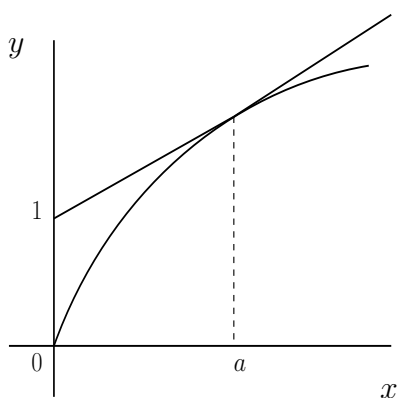
$$\lim_{x \rightarrow 2} \frac{(x-2) \sin(x-2)}{2 - 2 \cos^2(x-2)}.$$

**(III)** What is the area of the largest rectangle whose base is on the x-axis and whose upper corners are on the curve  $y = 12 - x^2$ .

**Chapter 2 Proficiency Test - Problem 2**

(I) Use the limit definition of the derivative to find  $f'(x)$  given that  $f(x) = \sqrt{x+3}$

(II) The figure below shows the graph of  $f(x) = \sqrt{x}$  and its tangent line at  $x = a$ . Find the value of  $a$ .



(III) Find the antiderivative for the function

$$f(x) = 6x^5 + \frac{5}{x^{10}} + \frac{4}{\sqrt{x}} + \pi$$

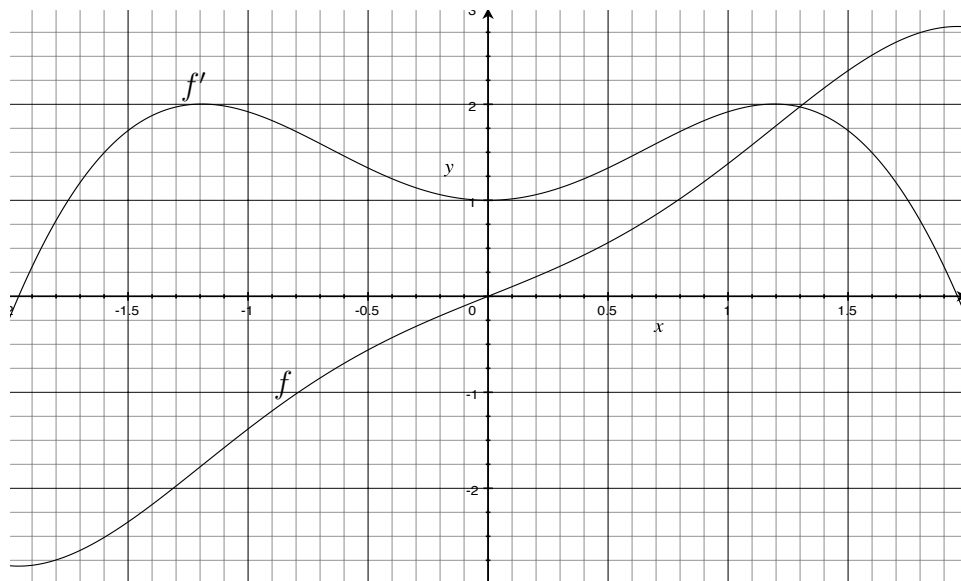
**Chapter 3 Proficiency Test - Problem 1**

(I) Let  $h(x) = A \cos(\sqrt{k} x) + B \sin(\sqrt{k} x)$ , where  $A$ ,  $B$  and  $k > 0$  are constants. Find values of  $A$ ,  $B$ , and  $k$  so that  $h''(x) = -4h(x)$ ,  $h(0) = 2$ , and  $h'(0) = 8$ .

(II) A tank of water initially contains 10 pounds of salt, thoroughly mixed. Pure water is added at the rate of 5 gallons per minute and the mixture is drained off at the same rate. The amount of salt in the tank at time  $t$ ,  $S(t)$ , decays at a rate modeled by the IVP  $S'(t) = -0.05S(t)$ ,  $S(0) = 10$ . Find  $S(t)$  and how much salt is left in the tank after 1 hour.

### Chapter 3 Proficiency Test - Problem 2

(I) The graph of the function  $f$  and its derivative  $f'$  are shown below.



$$\text{Let } h(x) = f(x) - x + \cos(f(x)), \quad m(x) = e^{f(x)^2}, \quad p(x) = \frac{\ln(\cos(f(x)))}{e^x + f(x)}.$$

(a) Show that  $h$  has a stationary point at  $x = 0$ .

(b) What is the concavity of  $h$  at  $x = 0$ ?

(c) Is  $m$  increasing at  $x = -1.2$  ?

(d) Show that  $p$  has a stationary point at  $x = 0$

(II) Find the equation of the tangent line to the curve  $y = 2 \sin(\pi x - y)$  at the point  $(1, 0)$ .

**Chapter 4 Proficiency Test - Problem 1**

Rewrite

$$\cos(\operatorname{arccot}(e^x/2))$$

as an algebraic expression. Be sure to draw the right angle triangle and label all the sides correctly.

**Chapter 4 Proficiency Test - Problem 2**

(I) Find  $\lim_{x \rightarrow 0^+} \arctan(x \ln x)$

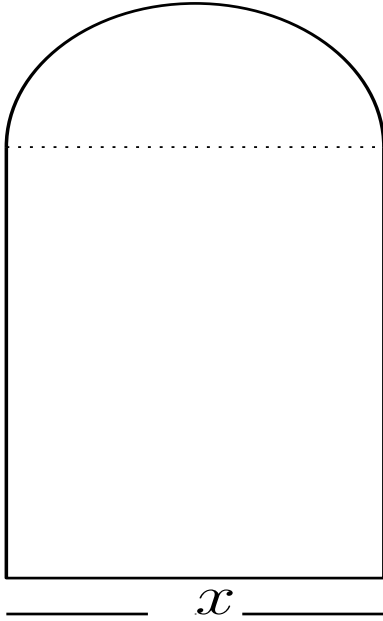
(II) Find  $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 + 3}$

(III) Find  $\lim_{x \rightarrow 0} \frac{\cos x}{\tan x}$



**Chapter 4 Proficiency Test - Problem 3**

A Norman window as depicted in the figure below has the shape of a rectangle surmounted by a semicircle. Suppose the outer perimeter of such a window must be 600 cm and the base length is  $x$  cm. Find the exact base length which maximizes the area of the window.



**Chapter 5I Proficiency Test - Problem 1**

A kite 50ft above the ground moves horizontally at a speed of 8ft/s. At what rate is the angle between the string and the horizontal decreasing when 200ft of string has been let out?

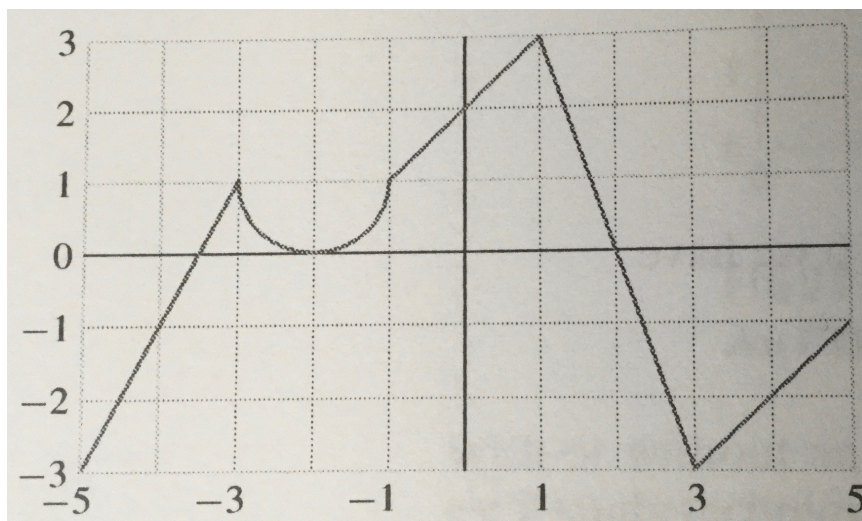
**Chapter 5I Proficiency Test - Problem 2**

(I) Use the intermediate value theorem to find an interval in which  $f(x) = x^5 + x^3 + 5x + 2$  has a root.

(II) Let  $f(x) = x^7 - x^5 - x^4 + 2x + 1$ . Is there a point  $c$  in the interval  $(-1, 1)$  at which  $f'(c) = 2$ ?

**Chapter 5I Proficiency Test - Problem 3**

(I) Use the graphs of  $f$  shown below to evaluate the integrals. [NOTE: The graph of  $f$  consists of four straight lines and a half circle.]



(a) Evaluate  $\int_{-3}^{-1} f(x) dx$

(b) Evaluate  $\int_5^0 4f(x) dx$

(II) Evaluate the integral

$$\int_0^{2\pi} \left( \sqrt{4\pi^2 - x^2} + \sin(x) \right) dx$$

(III) Suppose that  $f$  is a continuous function. If the average value of  $f$  over the interval  $[0, 1]$  is 2 and the average value of  $f$  over the interval  $[1, 3]$  is 4, what is the average value of  $f$  over the interval  $[0, 3]$ ?

**Chapter 5II Proficiency Test - Problem 1**

$$\text{Let } F(x) = \int_0^{\ln x} \sqrt[3]{t^2 + 27} dt.$$

(a) Find the equation of the line tangent to the graph of  $F(x)$  at  $x = 1$ .

(b) What is the concavity of  $F(x)$  at  $x = 1$ ?

**Chapter 5II Proficiency Test - Problem 2**

(I) Evaluate the integral

$$\int_0^2 ( 3 x^2 + e^x - \cos x ) dx$$

(II) Suppose that  $g'$  is a continuous function, that  $g(-2) = 3$ , and that the average value of  $g'$  over the interval  $[2, 5]$  is 1. Find  $g(5)$ .

(III) Approximate the integral  $\int_{-2}^2 x^3 dx$  using the trapezoid rule with 4 equal divisions, i.e. compute  $T_4$ .