Math 105: Review for Final Exam, Part I

1. Consider the function $f(x) = \dfrac{3}{5-2x}$.

   (a) Is this function continuous on the interval $(-\infty, \infty)$? Explain.

   (b) Compute the average rate of change of f on $[2, 2.01]$.

   (c) Using the limit definition of the derivative, compute $f'(x)$.

   (d) Find the equation of the tangent line to f at $x = 2$.

2. Given that $f(0) = 2$, $g(0) = 3$, $f'(0) = 5$, $g'(0) = 7$, and $f'(3) = \pi$ compute the following.

   (a) $h'(0)$ if $h(z) = f(z)g(z)$

   (b) $j'(0)$ if $j(z) = \dfrac{f(z)}{g(z)}$

   (c) $k'(0)$ if $k(z) = f(g(z))$

3. (a) Find $\dfrac{dy}{dt}$ if $y = t^5 + 5^t + e^5 + \dfrac{t}{5} + \dfrac{5}{t} + \dfrac{5}{\sqrt[5]{t}} + \ln(5t) + \arctan(5t) + \ln(5) + \sin 5$.

(b) Find $\dfrac{dy}{dx}$ if $y = \sqrt[3]{x}\cos(7x^3)$.

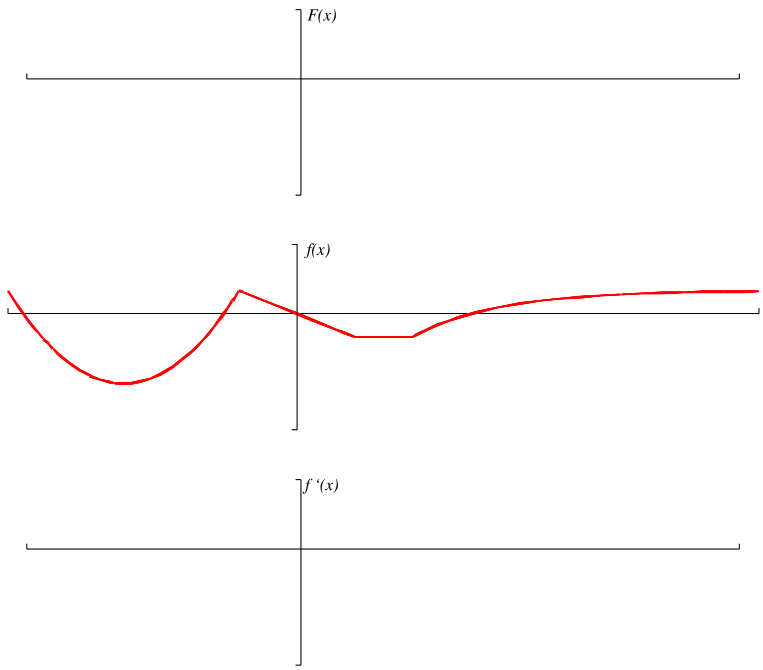
(c) Find $\dfrac{dy}{dz}$ if $y = \dfrac{e^z + e^\pi}{\tan 4 - 7z}$.

(d) Find $\dfrac{dy}{dr}$ if $y = \tan\left(e^{r^2 \arcsin(5r)}\right)$.

(e) Find $\dfrac{dy}{dx}$ if $y^3 + yx^2 + x^2 = 3y^2$.

(f) Find $\dfrac{dy}{dx}$ if $y = (1 + x^6)^{8x}$.

4. Given the graph of f, sketch a graph of f' and a graph of F, an antiderivative of f such that $F(0) = -1$.



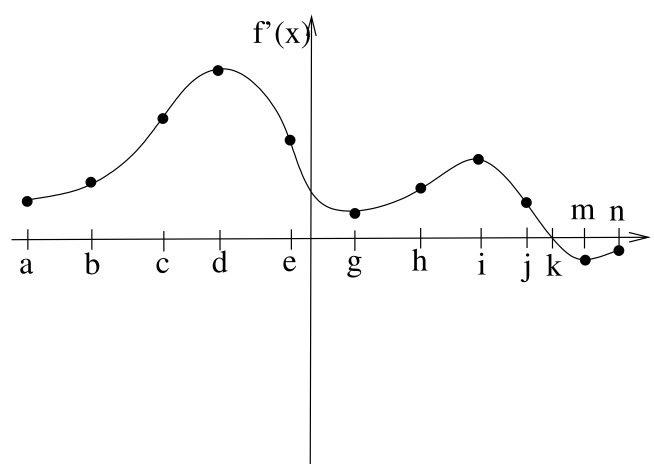
5. Shown below is a graph of f' on its entire domain. The graph is NOT f.

At which x-value(s) (if any)

(a) does f have a stationary point?
(b) does f have a local max?
(c) does f have a local min?
(d) does f' have a stationary point?
(e) does f' have a local max?
(f) does f' have a local min?
(g) is f greatest?
(h) is f least?
(i) is f' greatest?
(j) is f' least?
(k) is f'' greatest?
(l) is f'' least?

On what interval(s) is

(a) f increasing?
(b) f decreasing?
(c) f' increasing?
(d) f' decreasing?
(e) f concave up?
(f) f concave down?



6. Solve the IVP $y' = e^x - \sin x + 5$ given that $y(0) = 3$.

7. Evaluate the following limits.

(a) $\lim_{x \to \infty} \dfrac{x^2}{\ln x}$

(b) $\lim_{z \to 0} \dfrac{\sin(5z) - 5z}{z^3}$

(c) $\lim_{x \to 0} \dfrac{e^x - 1}{\cos x}$

(d) $\lim_{r \to 2} \dfrac{r^3 - 8}{r - 2}$

8. Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.

(a) Find the x- and y-coordinates of any and all critical points and use calculus to classify each as a local maximum, local minimum, or neither.

(b) Find the x- and y-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

(c) Find the x-coordinate(s) of any and all inflection points.