Name:______________________________

Mathematics 105
Sections A, B, C, and D
Final Exam
Dec 13, 2011

Problem	Possible	Actual
1	20	
2	15	
3	15	
4	15	
5	20	
6	15	
7	15	
8	15	
9	15	
10	15	
11	20	
12	20	
Total	200	

- You must show all work to receive credit.
- No electronic devices other than calculators are permitted.
- Give exact answers (such as $\ln 5$ or e^2) unless requested otherwise.
- Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers.
- Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit.
- If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

1. Consider the function:

$$g(x) = \begin{cases} x^3 - 1 & \text{if } x \geq 0 \\ 1 - x^3 & \text{if } x < 0 \end{cases}.$$

(a) What is $g'(x)$?

(b) Evaluate $\lim_{x \to 0^+} g(x)$.

(c) Evaluate $\lim_{x \to 0^-} g(x)$.

(d) Is $g(x)$ continuous at $x = 0$?

(e) Evaluate $\lim_{x \to 0^+} g'(x)$.

(f) Evaluate $\lim_{x \to 0^-} g'(x)$.

(g) In light of the above, check your answer to part (a). Does $g'(0)$ exist? Why or why not?

2. Evaluate $\int_{-1}^{1} \left(t^2 + \frac{1}{1+t^2} \right) dt$.

3. Find the solution to the initial value problem where $y' = x^2 + 2^x \ln 2$ with $y(0) = 2$.

4. Consider

$$f(x) = \begin{cases} a + bx^2 & \text{if } x < 2 \\ -x^2 + 10x - 4 & \text{if } x \geq 2 \end{cases}.$$

(a) What condition(s) must be placed on the constants a and b in order for f to be continuous on $(-\infty, \infty)$?

(b) For what values of the constants a and b will f be differentiable on $(-\infty, \infty)$?

5. A clock on the wall reads 10:00. The hour hand is $h = 5$ ft long and the minute hand is $m = 7$ ft long. The distance between to two tips is z. The angle between the two hands is θ.

   (a) The law of cosines states $h^2 + m^2 - 2hm\cos\theta = z^2$. Find z for this problem. [Hint: start by finding θ when the clock reads 10:00.]

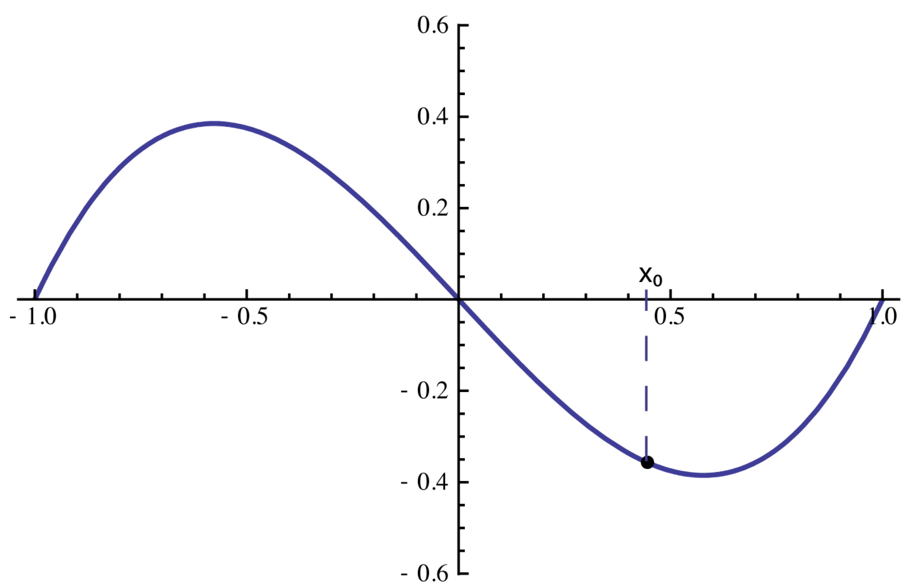
   (b) Explain why $\dfrac{d\theta}{dt} = \dfrac{11\pi}{6}$ radians per hour.

   (c) Find $\dfrac{dz}{dt}$.

6. Consider $f(x) = x^3 - x$, Newton's method generates successive estimates in finding a root of the equation $f(x) = 0$ using the formula $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ based upon an initial guess x_0.

(a) Starting with an initial guess of $x_0 = \frac{1}{\sqrt{5}}$, use Newton's method to generate the first four estimates, x_1, x_2, x_3, and x_4, to a root of $x^3 - x = 0$.

(b) Annotate the graph below to help demonstrate the geometric idea behind Newton's method for finding the estimates in part(a). You do not need to give a complete derivation of the formula, but you must informally describe how the method generates estimates.



7. Consider two functions,

$$f(x) = \int_{-3}^{x} \sin\sqrt{t}dt \text{ and } g(x) = \int_{3}^{x} \sin\sqrt{t}dt$$

(a) What is the derivative of $g(x)$?

(b) What is the derivative of $f(x)$?

(c) Why do your two answers differ?

8. Suppose $f(0) = 4$, $f'(0) = 3$, $g(0) = -7$, $g(4) = 1$, and $g'(0) = \frac{\pi}{2}$. Compute the following or explain why an answer does not exist.

(a) $h'(0)$ if $h(x) = f(x)g(x)$

(b) $k'(0)$ if $k(x) = \frac{f(x)}{g(x)}$

(c) $v'(0)$ if $v(x) = e^{g(f(x))}$

9. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose?

10. Consider the following limit

$$\lim_{x\to\infty} \frac{\sqrt{x+4}}{\sqrt{4x+1}}.$$

(a) What happens if you apply L'Hopital's Rule? Be sure to apply the rule more than once.

(b) Evaluate this limit algebraically.

11. This problem will evaluate an integral using the limit definition. We will evaluate

$$\int_0^1 x^3\,dx.$$

(a) If we partition the interval $[0,1]$ into n equal length subintervals, how long is each interval? This is Δx_i for all i.

(b) Write a formula for the right end-point of the i^{th} interval. This is c_i for all i.

(c) Write the integral as the limit of a sum.

(d) Using the fact that $\sum_{i=1}^{n} i^3 = \frac{n^4+2n^3+n^2}{4}$ evaluate the limit.

(e) Is your answer consistent with the answer the Fundamental Theorem of Calculus gives?

12. Consider a continuous function f defined on the interval $(0, \infty)$ with the following characteristics.

- $f(3) = 0$, $f'(2) = 0$, $f'(5) = 0$, $f'(3)$ is undefined, $f''(2) = 0$, and $f''(6) = 0$
- $f'(x) < 0$ on $(0, 2) \cup (2, 3) \cup (5, \infty)$, but $f'(x) > 0$ on $(3, 5)$
- $f''(x) > 0$ on $(0, 2) \cup (6, \infty)$, but $f''(x) < 0$ on $(2, 3) \cup (3, 6)$
- $\lim_{x \to \infty} f(x) = -3$ and $\lim_{x \to 0^+} f(x) = \infty$

It may help to do part (e) first.

(a) Identify all critical points for f. Classify each as a local maximum, local minimum, or neither.

(b) Identify all inflection points for the graph of f, if they exist. Justify your answer.

(c) Does f have a global maximum on $(0, \infty)$? If so, what is it?

(d) Does f have a global minimum on $(0, \infty)$? If so, what is it?

(e) Sketch a graph of f below.