MATH 205A,B - LINEAR ALGEBRA
FALL 2015

QUIZ 10

NAME: **Section:**(Circle one) A(8 : 00) B(9 : 30)

Show ALL your work CAREFULLY.

(a) Let

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 2 & 1 \end{bmatrix}.$$

Find an **orthonormal** basis for Col$A$.

Note that the columns of A form a basis for Col$A$**. Applying the Gram-Schmidt Process, let** $\vec{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ **and**

$$\vec{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \text{proj}_{\vec{x}_1} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} -1 \\ 7 \\ 4 \end{bmatrix}.$$

Normalizing the vectors $\vec{x}_1, \vec{x}_2$, we obtain the following orthonormal basis for Col$A$**:**

$$\left\{ \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{66}} \begin{bmatrix} -1 \\ 7 \\ 4 \end{bmatrix} \right\}$$

(b) Consider the matrix

$$B = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}.$$

Find B^{-1}. (Hint: is B orthogonal?)

A straightforward calculation shows that the columns of B form an orthogonal set. Moreover, each column vector has unit length so that B is an orthogonal matrix. It follows that

$$B^{-1} = B^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}.$$

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