

MATH106B,C CALCULUS II - PROF. P. WONG

FINAL EXAM - DECEMBER 11, 2013

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	18	
2.	18	
3.	18	
4.	22	
5.	14	
6.	24	
7.	18	
8.	18	
Total	150	

1. Evaluate each of the following integrals.

(9 pts.)(a)

$$\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}} dx$$

(9 pts.)(b)

$$\int x \sec^2 x dx$$

2. Evaluate each of the following integrals.

(9 pts.)(a)

$$\int \frac{2x + 1}{x^2 - 7x + 12} dx$$

(9 pts.)(b)

$$\int \frac{dx}{x^2 \sqrt{x^2 - 1}}.$$

3.(10 pts.)(a) Use the method of separation of variables to solve the following Initial Value Problem.

$$y' = 3x^2e^{-y}, \quad y(0) = 1.$$

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(8 pts.)(b) Consider the following given data of a function $f(x)$ on the interval $[0, 3]$.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	-1	0	1	3	1	-2	-4

Find M_3 (midpoint sum) and R_6 (right-hand sum).

4.(14 pts.) (a) Set up a definite integral representing the volume of the solid formed when the region bounded by the parabola $y = 2x - x^2$ and the line $y = x$ is revolved around the y -axis. [First sketch a picture of the region.]

(8 pts.) (b) Find the exact volume of the solid as in part (a) [i.e., evaluate the definite integral you found in part (a)].

5. (8 pts.)(a) Evaluate the following improper integral if it exists.

$$\int_{-1}^0 \frac{1}{\sqrt{1-x^2}} dx$$

- (6 pts.)(b) Write a definite integral [Do Not Evaluate] representing the arc length of the graph of $h(x) = \ln(\sec x)$ over the interval $[0, \frac{\pi}{4}]$. Simplify the definite integral as much as possible.

6. Determine whether each of the following series converges or diverges. **Justify your answer.**

(8 pts.)(a) $\sum_{n=0}^{\infty} \left(\frac{2^n + 1}{3^n} \right)$

(8 pts.)(b) $\sum_{j=1}^{\infty} \frac{\sqrt{j}}{j^2 + 1}$

(8 pts.)(c) $\sum_{k=1}^{\infty} \frac{1}{(\ln 2)^k}$

7.(10 pts.)(a) Let $f(x) = \sin(3x)$. Use the Maclaurin series for $\sin x$ to find $f^{(101)}(0)$ and $f^{(60)}(0)$.

(8 pts.)(b) Find a series representation of $(x - 1)e^{x^2}$. [Be sure to state the interval of convergence.]

8.(10 pts.)(a) Find the radius and the interval of convergence for the following power series. [Don't forget to check the endpoints.]

$$\sum_{n=1}^{\infty} \frac{nx^n}{n+1}$$

(8 pts.)(b) Determine whether the following series converges absolutely, conditionally, or neither.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1+n}{n^2} \right)$$