Math 106D **Grand Finale** **Name:**_____

Show all steps.
You do not need to simplify your arithmetic.

1. (8 pts.) Evaluate $\displaystyle\int_2^5 x^2\sqrt{x^3+1}\,dx$.

2. (8 pts.) Evaluate $\displaystyle\int x\,\ln(5x)\,dx$.

3. (8 pts.) Evaluate $\displaystyle\int \sec^{42}x\,\tan x\,dx$.

4. (10 pts.) Let $I = \displaystyle\int_1^9 e^{-x^2}\,dx$.
(a) Write out the right-hand Riemann sum approximation R_4 for I.
(b) Is R_4 an underestimate for I, an overestimate for I, or do we not have enough information to tell? Justify your answer.

5. (8 pts.) The base of a solid object is the region bounded between the curves $y = x^3$, $x = 0$, and $y = 8$. Cross-sections perpendicular to the x-axis are squares. Set up (but do not evaluate) an integral that computes the volume of the object.

6. (12 pts.) Let $f(x) = \sqrt{x}$.
(a) Find the second-order Taylor polynomial P_2 with base point $x_0 = 100$ for f.
(b) Apply our usual error formula to find a bound on the size of the error in using P_2 to approximate f on the interval $[95, 105]$.

7. (6 pts.) Use the comparison test to determine the convergence/divergence of the improper integral $\displaystyle\int_1^\infty \frac{x^5}{x^6+x+1}\,dx$. Clearly indicate the comparison you are using, and whether it's an underestimate or overestimate. (Make sure to use an estimate appropriate for the conclusion that you reach.)

8. (12 pts.) Let $a_k = \ln(2+\frac{1}{k}) - \ln(2+\frac{1}{k+1})$.
(a) Does the *sequence* $\{a_k\}_{k=1}^\infty$ converge? If so, find its limit. If not, explain why not.
(b) Does the *series* $\displaystyle\sum_{k=1}^\infty a_k$ converge? If so, evaluate it. If not, explain why not.

9. (6 pts.) Does the series $\frac{2}{1} - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \frac{7}{6} + \cdots$ converge? Explain why or why not.

10. (12 pts.) For the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(5x)^{2k}}{k}$,
(a) find the *radius* of convergence;
(b) find the *interval* of convergence.

11. (8 pts.) Using already-known series, find the Maclaurin series for $f(x) = \dfrac{x}{1-2x^3}$.

Optional bonus question. (5 pts.) If θ satisfies the differential equation $\dfrac{d^2\theta}{dt^2} = t + \theta$ with initial conditions $\theta(0) = 0$ and $\theta'(0) = 1$, find the coefficients a_0, a_1, a_2, a_3, a_4, a_5 in $\theta = \displaystyle\sum_{k=0}^{\infty} a_k t^k$.