

Name: _____

Math 105: Fall 2012
Final Exam

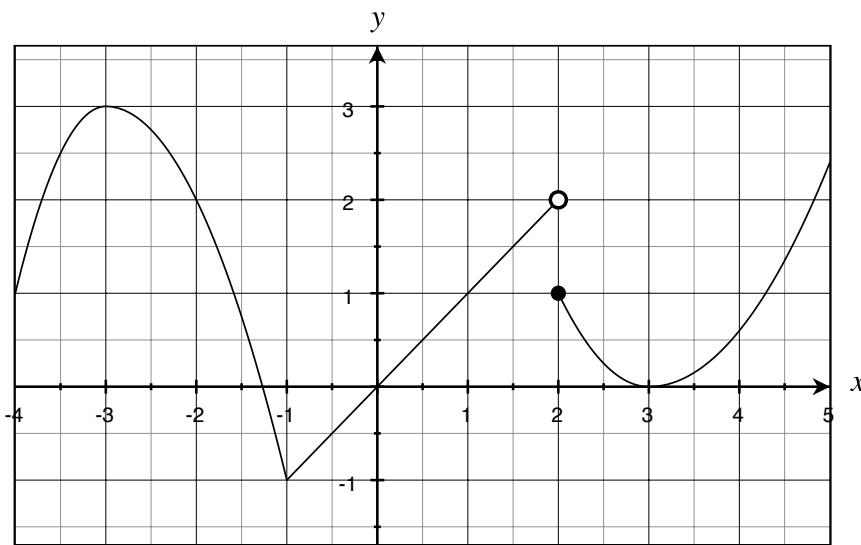
Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

Good Luck!

Formulas for Common Geometric Shapes

- Circle: $A = \pi r^2$, $C = 2\pi r$
- Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$
- Cylinder: $V = \pi r^2 h$, lateral area = $2\pi r h$

1. (12 points) The following is a graph of $f(x)$ on the interval $[-4, 5]$.



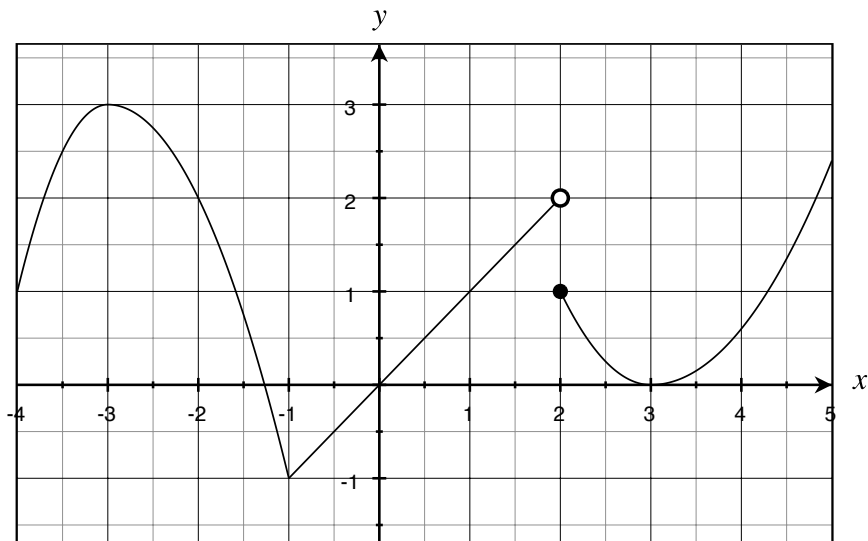
(a) Find $\lim_{x \rightarrow 2^-} f(x)$.

(b) Compute $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$.

(c) Use the graph to visually estimate $f'(-2)$.

(d) Let $A(x) = \int_{1/2}^x f(t) dt$. Compute $A(1.5)$.

2. (12 points) The following is a graph of $f(x)$ on the interval $[-4, 5]$.



(a) Let F be an antiderivative of f .

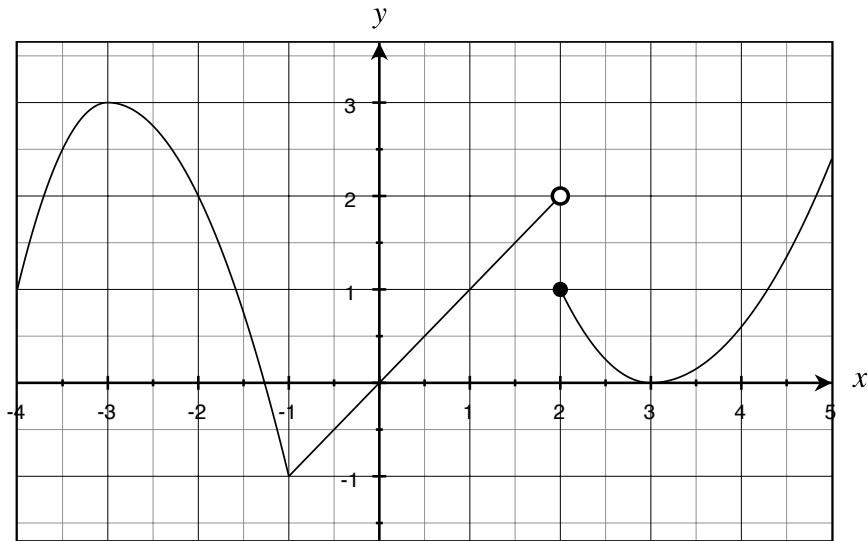
- Find the critical points of F .

- Find the inflection points of F .

(b) On what interval(s) is $f''(x)$ negative?

(c) On what interval(s) is $f'(x)$ decreasing?

3. (8 points) The following is a graph of $f(x)$ on the interval $[-4, 5]$.



(a) Do the hypotheses of the Mean Value Theorem hold for f on the interval $[-4, 1]$? Explain.

(b) Does the conclusion of the Mean Value Theorem hold for f on the interval $[-4, 1]$? Explain.

4. (20 points) Find the derivatives for each of the following.

(a) $f(x) = \sqrt[3]{x + \sqrt{x^e + e^\pi}}$. [You do NOT need to simplify your answer.]

(b) $z(t) = \frac{2^{\ln t}}{e^{-t^2}}$ [You do NOT need to simplify your answer.]

(c) $g(x) = \int_3^x 7 \arcsin t \, dt$ [You do NOT need to simplify your answer.]

(d) $y = x^{\sin x}$. [You do NOT need to simplify your answer.]

5. (5 points) Find $\frac{dy}{dx}$ if $xy + 5 = \frac{x}{2} - 3 \arctan y$.

6. (5 points) Compute $\int_0^\pi (\sin x + e^x + x^2) dx$.

7. (9 points) Choose **ONE** of the following. If you do more than one, then clearly mark which one you want graded. If you don't, the WORST will be chosen for you.
- (a) You are standing at the edge of a swimming pool and watching a friend who is swimming in a swim meet. Your friend is swimming in a straight line in a lane 5 feet from you. Your friend passes you and continues to swim at a speed of 4 feet per second. At the moment when your friend is 13 feet from you, at what rate is the distance between you and your friend changing? Be sure to express your answer using appropriate units.
- (b) A stone dropped into a still pond sends out a circular ripple. The radius is increasing at a rate of 3 ft/sec. After 10 seconds, what is the rate of change of the area? Be sure to express your answer using appropriate units.

8. (9 points) Use calculus to find the maximum possible volume of a cylinder if its total surface area (including both circular ends) is 150π . Be sure to confirm, using calculus, whether you have really found the maximum.

9. (20 points) Let $f(x) = x^2 - 7$.

(a) Estimate $\int_2^5 f(x) dx$ using L_3 , i.e., 3 rectangles and left-hand sums.

(b) Determine $\int_2^5 f(x) dx$ using the FTC.

(c) Use the limit definition of the definite integral to compute $\int_2^5 f(x) dx$, where $f(x) = x^2 - 7$.

$$\sum_{i=1}^n 1 = n \qquad \sum_{i=1}^n i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$