NAME_____

December 11, 2012   Mathematics 105D   Mr. Haines
                                    Calculus I
                              Final Examination

(5) I. Give an example of a function that is neither even nor odd and prove that
        your function is neither even nor odd by substituting values.

(8) II. If possible, give two distinct solutions to the differential equation $y' = 2x$.
        If not possible, explain why not.

(15) III. If $f(x) = \int_0^x e^{-t^2} \, dt$

    A. The formula for $f'(x)$ is: _____

    B. The formula for $f''(x)$ is: _____

    C. $\lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = $ _____

(20) IV. Suppose $f(x) = \dfrac{x-1}{x-1}$. Evaluate the following if they exist. If not, say why not.

    A. $f(2) = $ _____

    B. $f'(2) = $_____

    C. $\lim_{x \to 1} f(x) = $ _____

    D. Is f continuous at $x = 1$? Why or why not?

(18) V. Compute $\frac{dy}{dx}$ for the following:

A. $y = e^{\pi}$

B. $y = \sqrt{sinx}$

C. $y = \frac{\ln x}{x}$

(7) VI. Does $y = e^{3t}$ solve $y'' + 2y' - 15y = 1$? Explain.

(15) VII. The graph of $y^3 + yx^2 + x^2 = 3y^2$ is a complicated curve.

   A. Prove that $(1, 1)$ is a point on this curve.

   B. What is $\frac{dy}{dx}$ at the point $(1,1)$?

   C. What is the equation of the tangent line to this curve at the point $(1,1)$?

(12) VIII. You are seated on the top of a post 6 feet high. Your very cooperative dog is attached to the other end of the rope on a skateboard. You are pulling the rope toward you at a rate of 3 feet per minute. How fast is the dog moving toward the base of the post when the rope is 10 feet long?

(15) IX. Find the following limits:

A. $\lim_{x \to \infty} (x - 4)(9 - x)$

B. $\lim_{x \to \infty} \dfrac{x^2}{lnx}$

C. $\lim_{x \to 0} \dfrac{sin2x - 2x}{x^3}$

(10) X. Suppose that $y = f(x) = \ln(e^{x^2})$.

A. $f'(x) =$

B. $f''(x) =$

(25) XI. Suppose $f'(x) = x^3(x-2)(x+2) = x^5 - 4x^3$.

    A. Find all the stationary points of f .

    B. If $f(0) = 0$ give a formula for $f(x)$.

    C. Find all of the local maximum values of f on the interval $[-4,4]$.

    D. Give the intervals where f is decreasing.

    E. How many inflection points does f have?

(8) XII. Suppose $f''(x) = 3$ for all numbers x . Give a formula for $f(x)$
    assuming you know that $f(0) = 2$ and $f'(2) = 12$.

(12) XIII. What is the minimum value of the expression $x^2 + y^2$ if $2x + y = 5$?

(15) XIV. Suppose that $f(x) = \arcsin x$
    A. Give a formula for $f'(x)$.

    B. What is the value of $f'(0)$?

    C. Give the equation of the tangent line to the graph of $y = \arcsin x$ at the
       point $(0, f(0))$.

(15) XV. Suppose $f(x) = \frac{1}{x^2}$.

    A. What is the value of $\int_1^5 f(x)dx$?

    B. Draw a graph and compute the value of L_2 , the left sum approximation of the integral in part A with two rectangles.

    C. Draw a graph and compute the value of M_2 , the mid-point sum approximation of the integral in part A with two rectangles.