

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

1. (16 pts) Find derivatives for each of the following:

(a) $f(x) = \sqrt[3]{1 + \sqrt{x^e + e^\pi}}$ [Do not simplify your answer.]

(b) $z(t) = \frac{2^{\ln t}}{e^{-t^2}}$ [Do not simplify your answer.]

(c) $y = x^x$ [Simplify your answer!]

(d) Find $\frac{dy}{dx}$ if $xy + 5 = \frac{x}{2} - 3 \arctan y$

2. (8 pts) Give *exact* answers for each of the following:

(a) $\lim_{\theta \rightarrow 0} \frac{\tan(\pi\theta)}{3\theta}$

(b) Use the FTC to evaluate: $\int_1^3 \frac{\sqrt{w} + w}{w^{3/2}} dw$

3. (5 pts) Find the solution to the initial value problem where $y' = \sin x - 5e^x + 3x^2$ with $y(0) = 3$.

4. (5 pts) Let $f(x) = 3x - 5$. Use the limit definition of the derivative to show $f'(2) = 3$.

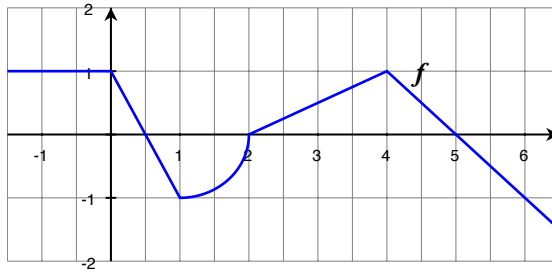
5. (8 pts) Consider $f(x) = \begin{cases} b - x^2, & \text{if } x < 3 \\ ax, & \text{if } x \geq 3 \end{cases}$

(a) What condition(s) must be placed the constants a and b in order for f to be continuous on $(-\infty, \infty)$?

(b) For what values of the constants a and b will f be differentiable on $(-\infty, \infty)$?

6. (8 pts) A 13-ft ladder is leaning against a house when its base starts to slide away from the wall. When the base of the ladder is 12 ft from the house, the base is moving at a rate of 5 ft/sec. At what rate is the angle θ between the ladder and the ground changing at that time?

7. (15 pts) Consider the graph of f given below. (Note: f is made of a circular segment and straight lines.)



(a) Consider the area function $F(x) = \int_{-1}^x f(t) dt$. Complete the table of values for the function $F(x)$.

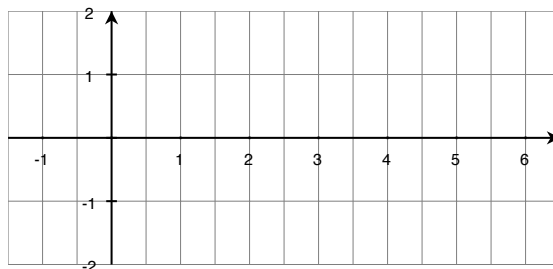
x	0	0.5	1	4	6
$F(x)$					

(b) Find $F'(x)$.

(c) Does the graph of F have critical points? On what interval(s) is F increasing? On what interval(s) is F decreasing?

(d) Does the graph of F have inflection points? On what interval(s) is F concave up? On what interval(s) is F concave down?

(e) Using the values from the table, sketch the graph of F . Be sure to label any local extrema and inflection points.



8. (10 pts) An outdoor track is to be created in the shape of a rectangle with semicircles attached on two opposite ends of the rectangle. The track must have a perimeter of 440 yards. Find the dimensions for the track that maximize the area of the *rectangular* portion of the field enclosed by the track.
- (a) What quantity are you trying to optimize? Are you trying to minimize it or maximize it?
- (b) Draw a picture and label the variables.
- (c) Write the objective function for the quantity you are trying to optimize.
- (d) Write the constraint equation(s) and use it to rewrite the objective function from (c) as a function of one variable.
- (e) Differentiate your objective function and find its critical point(s). Be sure to verify that your critical point optimizes the objective function.
- (f) What are the optimal dimensions for the track?

9. (13 pts) Evaluate $\int_2^4 (3x + 5) dx$ by computing the limit of right sums R_n . To do this, address each of the following:

(a) Partition the interval $[2, 4]$ into n equal length subintervals to find Δx , the width of each subinterval.

(b) Write a formula for the sampling point x_i , of the i^{th} interval.

(c) Write an expression that represents the right sum R_n .

(d) Write the integral as the limit of the sum. Evaluate this limit. You may find one of the following special sums useful:

$$\sum_{i=1}^n 1 = n, \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \text{and} \quad \sum_{i=1}^n i^2 = \frac{n(n-1)(2n+1)}{6}.$$

(e) Is your answer consistent with the answer found by applying the Fundamental Theorem of Calculus? In other words, check your answer by using the FTC to find $\int_2^4 (3x + 5) dx$.

10. (12 pts) The following may be either ‘True,’ ‘False,’ or somewhere ‘In Between’. Indicate ‘T’, ‘F’, or ‘IB,’ giving a *brief* explanation of your answer. You may include a drawing as part of your explanation, but you must also use words. Points are awarded only for the explanation—an unexplained ‘T’, ‘F’, or ‘IB’ gets no credit.

(a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

(b) If $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$ then f is continuous at $x = 2$.

(c) If $f''(a) = 0$ then f has an inflection point at $x = a$.

(d) If $f''(x) < 0$ on an interval I , then $f'(x)$ is increasing on that interval.

(e) If f is continuous for $a < x < b$ then f must have a maximum between a and b .

(f) If $f(1) < 0 < f(-2)$ then f must have a root on the interval $[-2, 1]$.

(g) If $\lim_{x \rightarrow \infty} f(x) = 2$ then the graph of f has vertical asymptote at $x = 2$.

(h) If $f(x) = x^2$ then $f'(2) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$.

(i) The definite integral $\int_2^3 \sqrt{x} dx$ may be found by computing $\lim_{n \rightarrow \infty} L_n$, where $L_n = \frac{1}{n} \sum_{k=0}^{n-1} \sqrt{2 + \frac{k}{n}}$ represents a Left Riemann Sum with n equal subintervals on $[2, 3]$.

BONUS: Find differentiable functions $f(x)$ and $g(x)$ such that $\lim_{x \rightarrow 3} f(x) = 0$, $\lim_{x \rightarrow 3} g(x) = 0$, and $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)} = \infty$.

Useful Formulas

- Formulas for Common Geometric Shapes

- Circle: $A = \pi r^2$, $C = 2\pi r$

- Trapezoid: $A = \frac{1}{2}b(h_1 + h_2)$

- Circular Cone: $V = \frac{1}{3}\pi r^2 h$

- Sphere: $V = \frac{4}{3}\pi r^3$, Surface Area $A = 4\pi r^2$

- Circular Cylinder: $V = \pi r^2 h$

- Log Properties

- $\ln(xy) = \ln x + \ln y$

- $\ln \frac{x}{y} = \ln x - \ln y$

- $\ln x^y = y \ln x$

- Special Sums

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n-1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$