FINAL

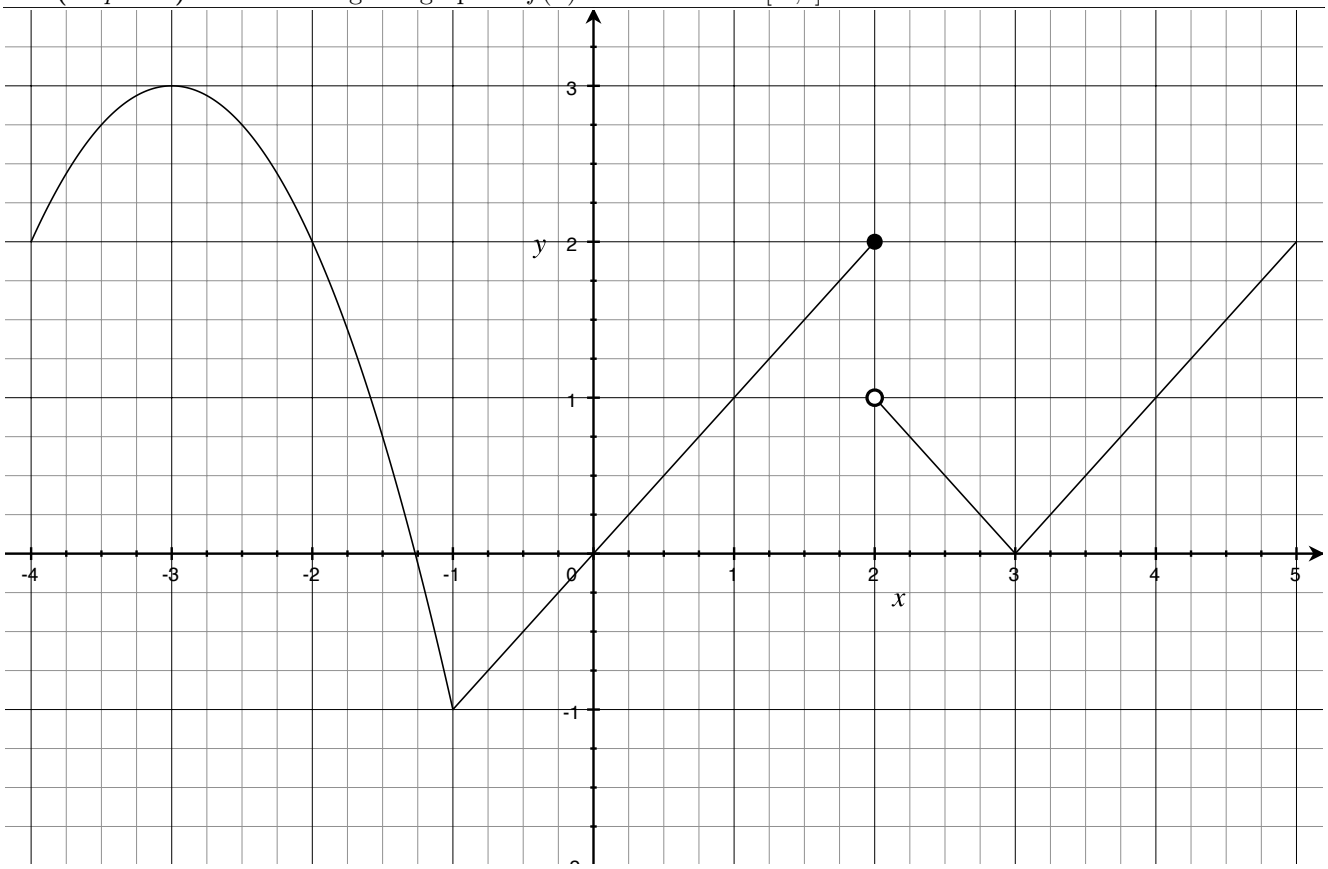
Math 105
12/11/12

Name: $\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*15 points*) The following is a graph of $f(x)$ on the interval [-4,5].



a. (*2 pts*) Where is $f'(x)$ positive?

b. (*2 pts*) Where is $f'(x)$ decreasing?

c. (*2 pts*) Where is $f''(x)$ negative?

d. (*2 pts*) Where is $f'(x)$ negative?

e. (*2 pts*) Does the hypothesis of the Mean Value Theorem hold on the interval [-4,2]? Explain.

f. (*2 pts*) Does the conclusion of the Mean Value Theorem hold on the interval [-4,2]? Explain.

g. (*3 pts*) Does the conclusion of the Intermediate Value Theorem hold on the interval [-4,3]? Explain.

2. (*10 points*)

x	$f(x)$	$g(x)$	$j(x)$	$f'(x)$	$g'(x)$	$j'(x)$
-2	-1	1	-1	3	2	1
-1	1	3	2	-1	3	-2
0	2	1	1	-2	-2	2
1	3	1	-1	-1	3	1
2	-2	2	1	3	2	3
3	-1	1	-1	1	-2	2

a. (*5 pts*) $H(x) = \sqrt{f(x^2)} + ln(j(x))$. Find $H'(1)$.

b. (*5 pts*) $F(x) = \dfrac{e^x g(x)}{f(x)^3}$. Find $F'(2)$.

3. (*8 points*) Find the derivative of the following functions.

a. (*4 pts*) $g(s) = \sqrt[5]{s^3} + \dfrac{5}{s} + 2^{cos(s)} + \arctan(e^{3s} + 5\pi) + \ln(3)$

b. (*4 pts*) $y = \dfrac{(2x+4)^3(x^2-2)^3 e^{-3x}}{(x-4)^4(5x^3-1)^2}$ (Use logarithmic differentiation)

4. (*7 points*) For the equation $x^3 + y^3 = ln(xy) - 1$ use implicit differentiation to find $\frac{dy}{dx}$.

5. (*8 points*) Solve the following. Only use L'Hôpital's rule when appropriate. Show your work!!

a. (*4 pts*) $\lim_{x \to 0} \frac{\sqrt{4-x^2}-2}{x}$

b. (*4 pts*) $\lim_{x \to 0} x^x$

6. (*7 points*) Find differentiable functions $f(x)$ and $g(x)$ such that $\lim_{x \to 5} f(x) = 0$ and $\lim_{x \to 5} g(x) = 0$ and

a. (*3 pts*) $\lim_{x \to 5} \frac{f(x)}{g(x)} = 10$

b. (*4 pts*) $\lim_{x \to 5} \frac{f(x)}{g(x)} = \infty$

7. (*8 points*) All edges of a cube are expanding at the same rate. The surface area is changing at a rate of 12 cm^2/second when each edge measures 3cm. Determine how fast the volume of the cube is changes when the edges are 3cm. (This is a two step problem)

8. (*10 points*)

a. (*4 pts*) Determine the antiderivative of $2t + \dfrac{2t}{1+4t^4}$. Show your check.

b. (*2 pts*) Find the derivative of $\ln(32 - 5x^2)$.

c. (*4 pts*) Use the FTC to determine $\displaystyle\int_0^2 \frac{x}{32-5x^2}dx$.

9. (*10 points*) A right triangle is formed in the first quadrant by the $x-$ and y-axes and a line through the point (2,3). Find the vertices of the triangle so that its area is minimum. (Hint: The area of the triangle should be a function of the slope of the line. You don't know the slope.)

10. (*17 points*) Let $f(x) = 3x^2 + 1$.

a. (*4 pts*) Estimate the area $\int_1^4 f(x)dx$ using 3 rectangles and left-hand sums.

b. (*3 pts*) Determine $\int_1^4 f(x)dx$ using the FTC.

c. (*10 pts*) Use infinite Riemann sums $\left(\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x\right)$ to find $\int_1^4 f(x)dx$.

$$\sum_{i=1}^{n} 1 = n, \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$