

1. Find $\int \frac{3e^{3x}}{(\sqrt[4]{e^{3x}+1})^5} dx$; show any substitution you used.

Notation help: $\frac{1}{(\sqrt[4]{e^{3x}+1})^5}$ is $(e^{3x}+1)^{-5/4}$.

2. For any number $a \geq 0$, the improper integral $\int_a^\infty \frac{3e^{3x}}{(\sqrt[4]{e^{3x}+1})^5} dx$ converges. Show how this is true, and find what the integral converges to. (Your answer will be in terms of a). Show all your work and use good notation. You might find it useful to make a table to investigate some parts of the limit you'll be finding.

3A. Consider the series $\sum_{k=0}^{\infty} \frac{3e^{3k}}{\left(\sqrt[4]{e^{3k} + 1}\right)^5}$. Use the integral test to explain why this series must converge. Use good notation. You may use the results of problem 2 (previous page).

3B. Suppose this series converges to the number S . Find the value of N for which the partial sum $\sum_{k=0}^N \frac{3e^{3k}}{\left(\sqrt[4]{e^{3k} + 1}\right)^5}$ is guaranteed to be within $\epsilon = 1/100,000$ of S . Show all your work.

HINTS: Recall that $\sum_{k=N+1}^{\infty} a_k \leq \int_N^{\infty} f(x) dx$. Your answer to problem 2 will again be useful.

3C. For your value of N , use the LHS program to find the partial sum in 3B to at least 8 places after the decimal point. If you used a summation program that's on your calculator, say so; otherwise give the B , A and N required by the LHS program.

4A. Find the interval of convergence (IOC), endpoints pending, for the series $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k \cdot 4^k}$.

4B. Is the left-hand endpoint of the answer to 4A in the IOC or not? Explain your reason carefully.

4C. Is the right-hand endpoint of the answer to 4A in the IOC or not? Explain your reason carefully.

5A. Consider the series $2.2 - 2.02 + 2.002 - 2.0002 + 2.00002 - \dots$. Does it converge absolutely, conditionally, or does it diverge? Explain your answer completely.

5B. If the series in 5A converges, what does it converge to?

6A. Consider the series $\sum_{k=-2}^{\infty} 8 \left(\frac{-4}{5} \right)^k$. Write out the first 5 terms explicitly (you don't need to simplify them, but please note the " $k = -2$ " under the summation sign).

6B. You should recognize this as a geometric series. What are a and r ?

6C. Does the series converge? If so, to what does it converge?

7A. Write out the first five terms of the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k}}$.

(In your answer use no decimals; keep the $\sqrt{\quad}$'s).

7B. Explain: does the alternating series in 7A pass the alternating series test?

7C. Find the smallest N for which the partial sum $\sum_{k=1}^N (-1)^{k+1} \frac{1}{\sqrt{k}}$ is guaranteed to be within 0.03333333... of whatever the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k}}$ converges to.

8a. Use an appropriate substitution to find $\int_0^1 \frac{e^{2x} + x}{e^{2x} + x^2 + 2} dx$. (Show all steps).

8b. Don't miss this question: What are the new limits on the integral after you make your substitution?

9A. Find the Taylor polynomial $P_3(x)$ of degree 3 in powers of $(x - 1)$ for the function $f(x) = \sqrt[4]{x}$. Organize your work into the usual table as we've done in class.

9B. Find the "error guarantee" if the polynomial P_3 is used to approximate $\sqrt[4]{1.5}$. Show all the steps.

9C. What are $P_3(1.5)$ and $f(1.5)$, respectively?

9D. What is the actual error if $P_3(1.5)$ is used as an approximation to $\sqrt[4]{1.5}$? Is it less than the guarantee?

10. Show how to find $\int \frac{dx}{x^2\sqrt{x^2-1}}$ using the substitution $x = \sec t$. Show all your work, and of course, the triangle you need to “unsubstitute” at the end of the problem.

11A. Set up the integral that represents the arc length of $f(x) = x^3$ from $x = 2$ to $x = 3$.

11B. What is the contribution of the 4th term in the LHS(10) approximation of the integral you've produced in 11A?

12. Find $\int \frac{\ln x}{x^2} dx$.

13A. Give the first four non-zero terms of the Maclaurin series for $\cos w$. (Hopefully from memory, but derive it otherwise).

13B. Use the answer to 13A to find the first four non-zero terms of a Maclaurin series for $\cos(3t^2)$.

13C. Use the answer to 13B to find the first four non-zero terms of a Maclaurin series for $\int_0^x \cos(3t^2) dt$.

13D. What does your answer to 13C give if used to approximate $\int_0^1 \cos(3t^2) dt$? (Show to many places)

13E. Find the integral in 13D using MID(100) on your calculator. Does it come out close to the answer in 13D?

3F. Bonus! Let $f(t) = \cos(3t^2)$. Find $f^{(6)}(0)$ and $f^{(7)}(0)$. Show your work.