

Directions: Complete all of the following to the best of your ability. If you do not understand a question, please let me know; I may be able to assist you. SHOW ALL WORK! You will be graded primarily on the method you use, not your final answer. GOOD LUCK!

Name: _____

Question	Points Possible	Score
1	24	
2	12	
3	12	
4	20	
5	20	
6	12	
Bonus 1 & 2	5	
Total	100	

DO NOT WRITE IN THE BOXES ABOVE!

Final Exam Formula Sheet

- $e^{A+B} = e^A \cdot e^B$
- $e^{A-B} = \frac{e^A}{e^B}$
- $\cos(A \pm B) = \cos(A) \cos(B) \mp \sin(A) \sin(B)$
- $\sin(A \pm B) = \sin(A) \cos(B) \pm \cos(A) \sin(B)$
- $\ln(A \cdot B) = \ln(A) + \ln(B)$

1. Determine if the series is convergent or divergent by making a comparison (DCT or LCT) with a suitable b_n . Fill in the blanks with your answer. For “Convergent or Divergent” write “Convergent” or “C” if the series to the left is convergent. Otherwise write “Divergent” or “D”. Then write a sequence that could be used to make your comparison. You do not need to show any work.

Series	Convergent or Divergent?	b_n ?
(a) $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$	_____	_____
(b) $\sum_{n=1}^{\infty} \frac{3 + 2 \cos(n)}{n}$	_____	_____
(c) $\sum_{n=1}^{\infty} \frac{4^n}{2^n + 8^n}$	_____	_____

Answer the following questions about the series by filling in the blanks. For “Convergent or Divergent” write “Convergent” or “C” if the series to the left is convergent. Otherwise write “Divergent” or “D”. For “Test Used” write an appropriate series test that tells you whether the series is convergent or divergent. You can use test abbreviations. You do not need to show any work, although you may find it helpful to write the sums using series notation.

Series	Convergent or Divergent?	Test Used?
(d) $8 - 2 + \frac{1}{2} - \frac{1}{8} + \frac{1}{32} - \dots$	_____	_____
(e) $2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} + \dots$	_____	_____
(f) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{2} + \frac{1}{\sqrt{5}} + \dots$	_____	_____

2. Find the indicated Taylor Series. Simplify your answer.

(i) $f(x) = \ln(x)$; $a = 4$

(ii) $f(x) = \sin(2x)$; $a = \frac{\pi}{2}$

3. (a) Determine if the series $\sum_{n=1}^{\infty} \left(\frac{3n^2 - 3}{1 + 4n^2} \right)^n$ is absolutely convergent, conditionally convergent, or divergent.

- (b) What can you say about $\lim_{n \rightarrow \infty} \left(\frac{3n^2 - 3}{1 + 4n^2} \right)^n$? Support your answer with words.

4. (a) Use techniques of integration (not power series) to evaluate the integral $\int \frac{\cos(6\sqrt{x})}{\sqrt{x}} dx$.

(b) Now evaluate the integral $\int \frac{\cos(6\sqrt{x})}{\sqrt{x}} dx$ using power series.

5. (a) Evaluate the integral $\int 3x^2 \cdot \arctan(x) \, dx$ using power series.

(b) Now use techniques of integration (not power series) to evaluate the integral $\int 3x^2 \cdot \arctan(x) \, dx$.

6. (a) Find a range of x values for which the function is defined, then state the radius of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (3x+1)^n}{n}.$$

- (b) Is the input $x = 0$ valid in the function $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (3x+1)^n}{n}$? Explain your answer.

Bonus Questions

1. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$ is convergent or divergent.

2. Find the sum of the series. $\sum_{n=1}^{\infty} \frac{n}{3^n}$.