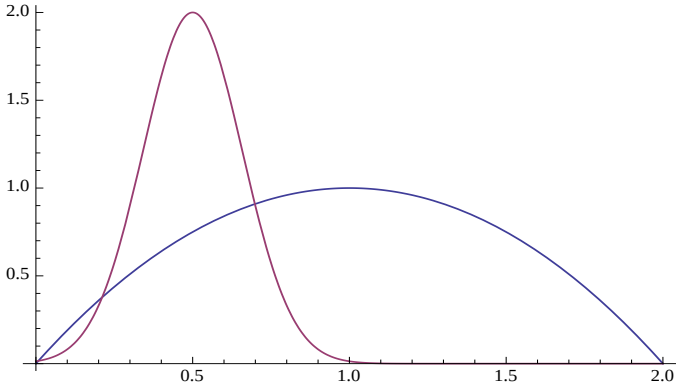
Name:_____

Final Exam

Show all your work to receive full credit for a problem. There are a total of 100 points on this test. Good luck!

1. (4 pts. each) Two children riding their bikes start at the same time and travel in the same direction along a straight road. The figure below gives the velocity, v, of each bike as a function of time, t.



   (a) Which bike attains the larger maximum velocity?

   (b) Which bike stops first?

   (c) Which bike travels farther?

2. (4 pts. each) Let f be a function which is continuous on $[-2, 2]$ and differentiable on $(-2, 2)$. The table below gives the values of f at some points in the interval.

x	$f(x)$
-2	2.5
-1.5	3
-1	-.5
-.5	-1
0	-2
0.5	-3
1	-2
1.5	-1.75
2	-1.5

(a) Explain why the function f must have a root, and find the smallest interval in which the function must have a root.

(b) Explain why there must be a point c in the interval $(-2, 2)$ at which $f'(c) = -4$.

(c) Explain why f must attain a maximum value, and estimate on which interval this maximum probably occurs.

3. (5 pts) Find the equation of the line tangent to the graph of $p(x) = \ln(e^{4x} + \sin(2x))$ at $x = 0$.

4. (5 pts) Find $\dfrac{dy}{dx}$ for $\sin^2(x) + \cos^2(y) = 1$.

5. (5 pts) Verify that $y = 5e^t + t^3$ is a solution to the DE given by $\dfrac{dy}{dt} = y + t^2(3 - t)$.

6. (4 pts each) Evaluate the following limits.

(a) $\lim_{x\to\infty} \dfrac{2x^2-7}{4x^3+x^2-1}$

(b) $\lim_{x\to 1} \dfrac{\ln x}{1-x}$

7. (5 pts. each) Evaluate the following integrals:

(a) $\int_1^9 (x^{5/2}+2x^{1/2})dx$

(b) $\int_{-\pi/2}^0 3\sin x dx$

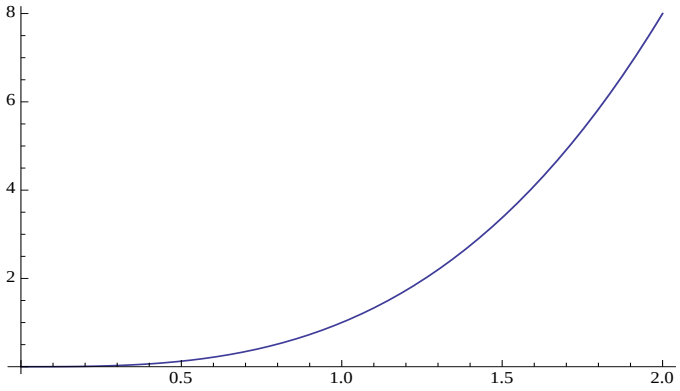
8. (4 pts) Which of the following correctly expresses the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$, as an integral? Briefly justify your answer in the space provided.

(a) $\int_0^1 x^2 dx$ (d) $\int_1^2 x^4 dx$

(b) $\int_1^2 x^3 dx$ (e) $\int_0^1 x^3 dx$

(c) $\int_0^1 x^4 dx$ (f) $\int_1^2 x^2 dx$

9. (4 pts each) Consider the function $h(x) = x^3$ on the interval $[0, 2]$. Below is the graph of h.

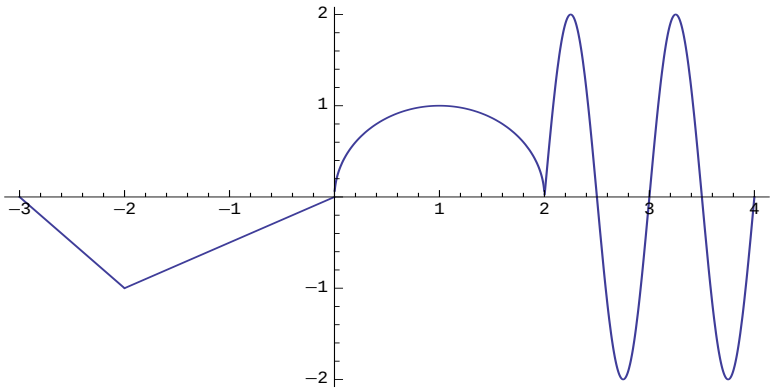


(a) Prove that $0 \leq \int_0^2 h(x)dx \leq 16$.

(b) Partition the interval into four equal subintervals and use left approximating sums to compute an estimate of $\int_0^2 h(x)dx$.

(c) Use the Fundamental Theorem of Calculus to calculate the exact value of the integral $\int_0^2 h(x)dx$. How do the three quantities compare to each other?

10. (3 pts each) Let $G(x) = \int_0^x g(t)dt$ where $g(t)$ is the function shown in the figure. Answer the following questions about G. (You may use estimates if you're not sure of the exact values.)



(a) What is $G(-2)$? $G(1)$?

(b) What is $G'(1)$? What is $G''(1)$?

(c) Where are the stationary points of G?

(d) On what intervals is G increasing? On what intervals is G decreasing?

(e) What are the inflection points of G?

(f) On what intervals is G concave up? On what intervals is G concave down?

11. (9 pts) A man 6 feet tall walks at the rate of 5 feet per second toward a street light that is 16 feet above the ground. At what rate is the tip of his shadow moving?