Name:____

Math 105: Fall 2013
Final Exam

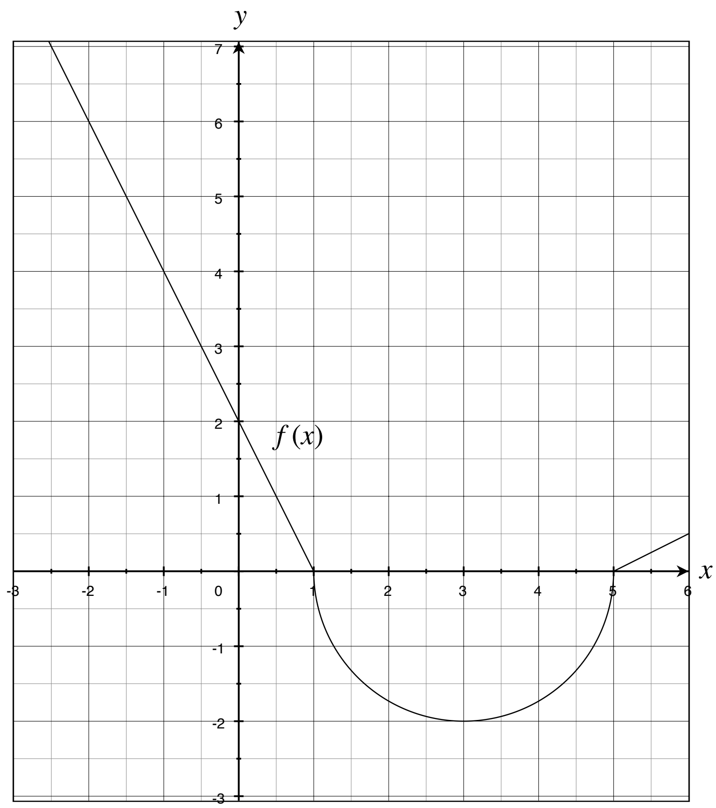
Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

Good Luck!

Formulas for Common Geometric Shapes • Circle: $A = \pi r^2$, $C = 2\pi r$ • Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

The Unit Circle is on the back of the last page.

1. (6 points) The following is a graph of $f(x)$ on the interval $[-3, 6]$. $f(x)$ consists of lines and a semicircle.



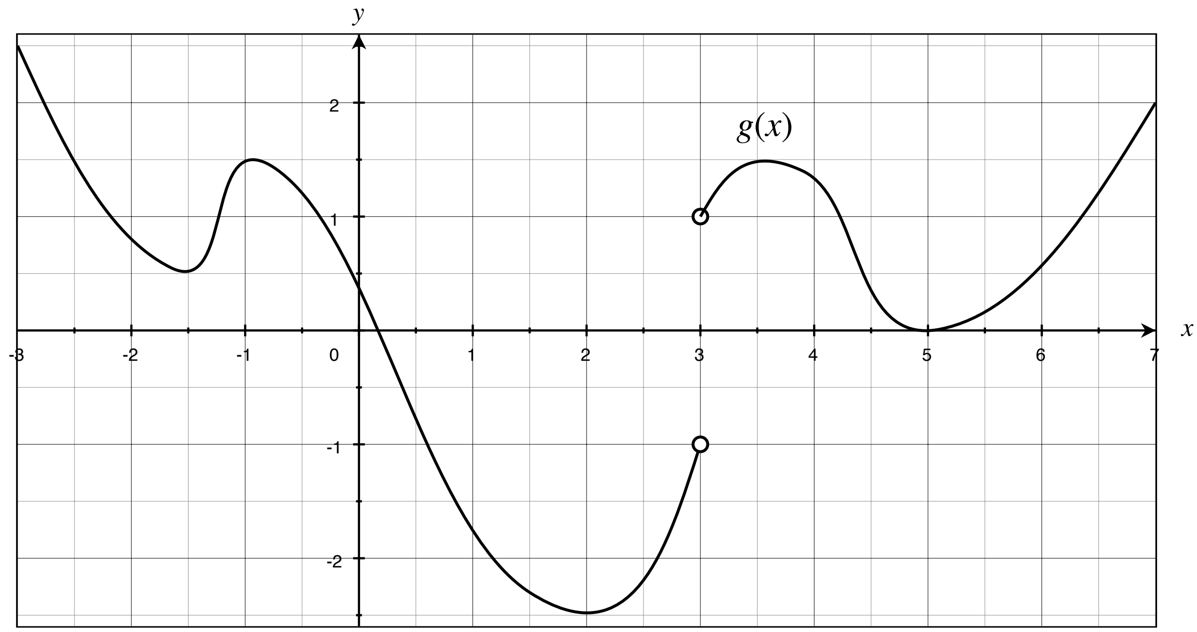
Let $A(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Compute $A(3)$.

(b) Compute $A'(-1)$.

2. (3 points) Suppose that $g(x)$ is a function with $g(1) = 2$, $g'(1) = -4$, and $g''(1) = 3$. Find the equation of the line tangent to the graph of $g(x)$ at $x = 1$.

3. (12 points) The following is a graph of $g(x)$ on the interval $[-3, 7]$. Let G be an antiderivative of g.



(a) Use the graph to visually estimate $g'(1)$. (Make it clear how you obtained your answer.)

(b) Find the x-values of the critical points of G.

(c) Classify your critical points from (b) as local maxima (of G), local minima (of G), or neither. (Be sure to show how you obtained your answer.)

(d) On what interval(s) is $G(x)$ concave down?

4. (9 points) Tell whether each of the following statements is <u>TRUE</u> or <u>FALSE</u>. BRIEFLY justify your answers.

(a) If $f'(x) > 0$, then the slope of f must be increasing.

(b) If $h''(x) = 6x + 4$, then $h(x)$ has an inflection point at $x = -2/3$.

(c) If the polynomial $p(x)$ satisfies $p(3) = 10$ and $p(4) = 5$, then the Intermediate Value Theorem (IVT) implies that p must have a root/zero somewhere between 3 and 4.

5. (4 points) Let $g(x) = \cos x$. Find the exact value of $\lim_{h \to 0} \frac{g(\frac{\pi}{3} + h) - g(\frac{\pi}{3})}{h}$.

6. (10 points) Find the derivatives for each of the following.

(a) $f(x) = \left(\frac{1}{x} - 21 \ln(3x + 4)\right)^{12}$ [You do NOT need to simplify your answer.]

(b) $y = \frac{\arcsin x}{5^x + 7}$ [You do NOT need to simplify your answer.]

7. (5 points) Compute $\int_1^2 \left(7e^x + \sqrt[3]{x}\right)\, dx$. Approximate your answer to 3 decimal places.

8. (5 points) Find $\dfrac{dy}{dx}$ if $x^3y + y^3 = 7 + x$.

9. (5 points) Verify that $y = 4e^{3t} + t^2$ is a solution to the differential equation

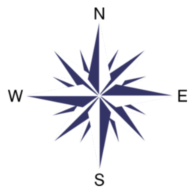
$$\frac{dy}{dt} = 3y + t(2 - 3t).$$

10. (8 points) Solve the following. Only use L'Hôpital's rule when appropriate. Show your work!

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x}$

(b) $\displaystyle\lim_{n\to 0} \frac{n^4}{4^n}$

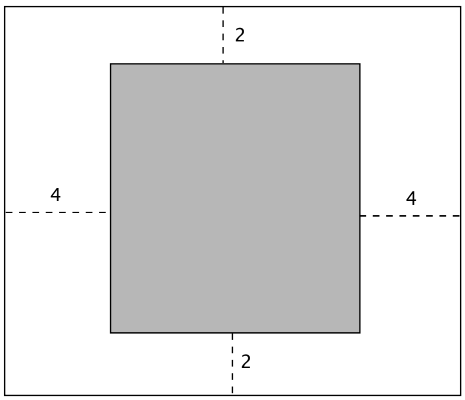
11. (8 points) Train A is 15 miles east of the station and moving east at 20 mph. Train B is 60 miles south of the station and moving north at 15 mph. At what rate is the distance between them changing? Is the distance between them increasing or decreasing?



12. (8 points) A poster will have

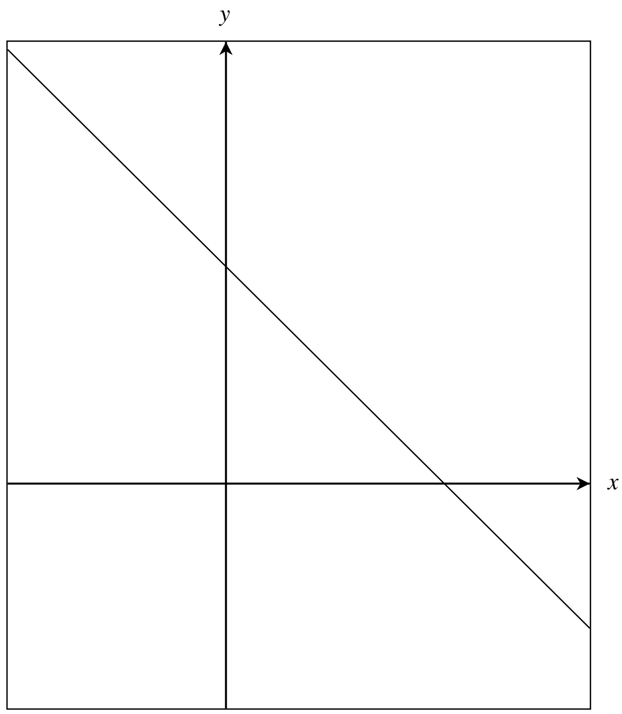
- 100 in^2 of pictures (shaded area)
- 2 inch margins on the top and bottom
- 4 inch margins on the left and right

(Diagram not to scale.) Find the dimensions that minimize the overall area. Be sure to confirm, using calculus, whether you have really found the minimum.



13. (10 points) Let $f(x) = -x + 3$.

(a) Estimate $\int_{-2}^{4} f(x)\,dx$ using R_3, i.e., 3 rectangles and righthand sums.

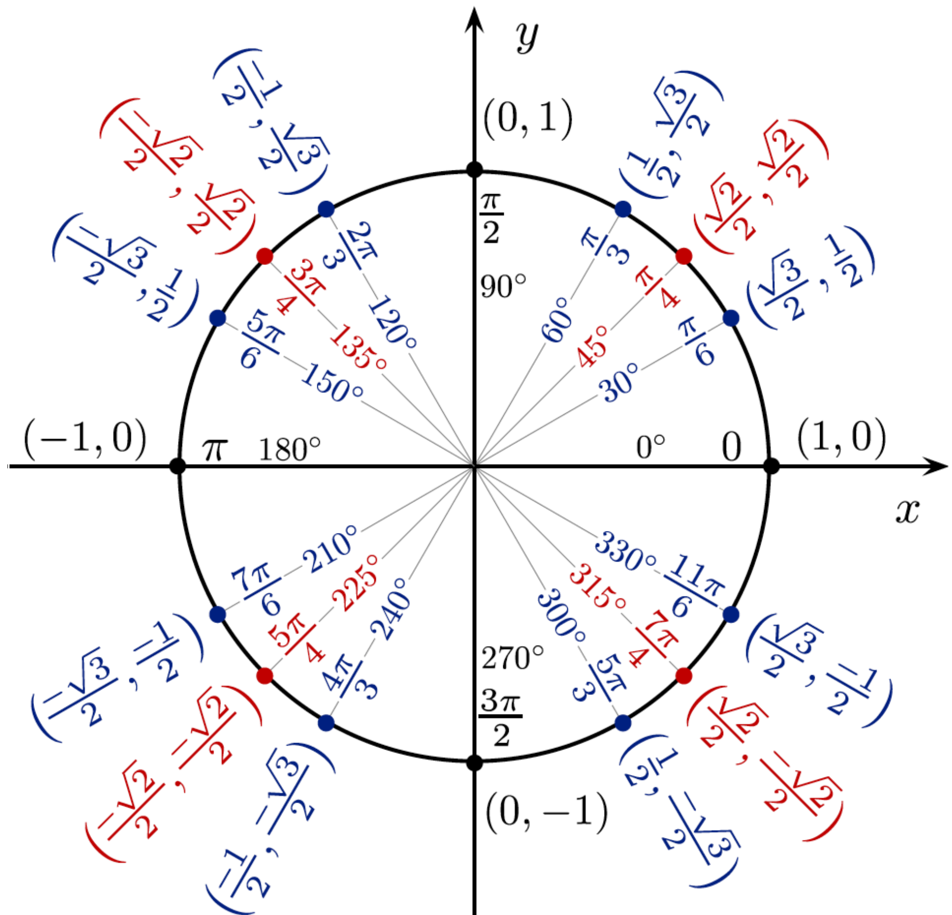


(b) Determine $\int_{-2}^{4} f(x)\,dx$ using the Fundamental Theorem of Calculus (FTC).

14. (7 points) In the following problem we will use the limit definition of the definite integral to compute $\int_{-2}^{4} f(x)\,dx$, where $f(x) = -x + 3$.

(a) Show that $R_n = \sum_{i=1}^{n} \frac{30}{n} - \frac{36}{n^2} i$.

(b) The result in part (a) simplifies to $R_n = 12 - \frac{18}{n}$. Use this fact to compute $\int_{-2}^{4} f(x)\,dx$ exactly.



y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
π/2
90°
0
0°
π
180°
3π/2
270°
π/6
30°
π/4
45°
π/3
60°
2π/3
120°
3π/4
135°
5π/6
150°
7π/6
210°
5π/4
225°
4π/3
240°
5π/3
300°
7π/4
315°
11π/6
330°
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
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