Math 105: Review for Final Exam, Part II

1. Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.

   (a) Find the x- and y-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

   (b) Find the x- and y-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

   (c) Find the x-coordinate(s) of any and all inflection points.

2. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is \$9.00 per container, what dimensions will give the largest volume?

   area of circle $= \pi r^2$ lateral area of cylinder $= 2\pi rh$ volume of cylinder $= \pi r^2 h$

3. You are standing on a pier, 6 feet above the deck of a boat. Attached to the boat is a line, which you are pulling in at a rate of 3 feet per second. When there are 10 feet of line between your hand and the boat, at what rate is the boat moving across the water?

4. Use the Intermediate Value Theorem to show that $f(x) = x^3 - 2x - 1$ has a root on $[1, 2]$.

5. What (if anything) does the Extreme Value Theorem say about $f(x) = x^2$ on each of the following intervals?

   (a) $[1, 4]$

   (b) $(1, 4)$

6. Find the value of the constant c that the Mean Value Theorem specifies for $f(x) = x^3 + x$ on $[0, 3]$.

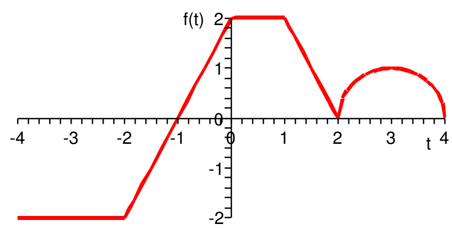
7. Water is leaking out of a tank at a decreasing rate $r(t)$ as shown in the table below.

time (min)	0	2	4	6	8
rate (gal/min)	15	11	8	4	3

(a) Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

(b) Interpret the expression $\int_2^6 r(t)\,dt$ in terms of the situation described above.

8. Consider the graph of $f(t)$ shown. It is made of straight lines and a semicircle.



Let $G(x) = \int_0^x f(t)\ dt$ and $H(x) = \int_{-3}^x f(t)\ dt$.

(a) Compute $G(2)$, $G(4)$, and $H(4)$.

(b) Where is G increasing? Where is G decreasing?

(c) Where is G concave up? Where is G concave down?

(d) At what x-value(s) does G have a local maximum? At what x-value(s) does G have a local minimum?

(e) Find a formula that relates G and H.

(f) How would your answers to (b), (c), and (d) change if the questions were about H instead of G?

9. (a) Use sigma notation to express L_{10} and M_{10} as approximations to $\int_{20}^{60} \ln x \, dx$.

(b) Draw a sketch that represents the sum M_4.

10. Find the following.

(a) all antiderivatives of $1 + 2x + x^3 + 4\sqrt{x} + \dfrac{1}{x^5}$

(b) $\int_{-2}^{2} \sqrt{4 - x^2} \, dx$

(c) $\dfrac{d}{dx} \int_{1}^{x} \sin \sqrt{t} \, dt$

(d) $\int_{0}^{2} x^2 \, dx$

Do this first with the limit definition of the definite integral then check your answer with the Fundamental Theorem.

You may use the fact that $\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$.