Math 106: Review for Final Exam, Part II

1. Use a second-order Taylor polynomial to estimate $\sqrt[3]{28}$.

2. What is the largest possible error that could have occurred in your previous estimate?

3. Use a comparison to show whether each of the following converges or diverges. If an integral converges, give a good upper bound for its value.

   (a) $\displaystyle\int_1^\infty \frac{7+5\sin x}{x^2}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{1+3x^2+2x^3}{\sqrt[3]{10x^{12}+17x^{10}}}\,dx$

4. Decide if each of the following sequences $\{a_k\}_{k=1}^{\infty}$ converges or diverges. If a sequence converges, compute its limit.

(a) $a_k = 3 + \dfrac{1}{10^k}$

(b) $a_k = (-1)^k$

(c) $a_k = \dfrac{5e^k}{7e^k + \ln(k+1)}$

Strategy. The following is a good order in which to consider the various series convergence tests.

(a) Do the individual terms approach 0? If they don't approach 0, the nth Term Test tells you the series must diverge. If they do approach 0, try another test.

(b) Is the series geometric? (That is, do you multiply by the same constant r to get from each term to the next?) If so, the series converges if $|r| < 1$ and diverges otherwise.

(c) Does the series contain something such as $(-1)^k$ or $(-1)^{k+1}$ or $\cos(k\pi)$ that makes its terms alternate? If so, try the Alternating Series Test.

(d) Does the series contain a factorial ($k!$) or exponential (such as 2^k or e^k)? If so, try the Ratio Test.

(e) If the series has positive terms, does it remind you of a simpler series (especially a p-series: powers of k such as $1/k$ or $1/k^2$)? If so, try the Comparison Test.

(f) Is the formula something you can integrate easily? If so, try the Integral Test.

5. Decide if each of the following series converges or diverges. If a series converges, find its value.

(a) $3.1 + 3.01 + 3.001 + 3.0001 + \cdots$

(b) $1 + 1/2 + 1/3 + 1/4 + \cdots$

(c) $5 - 5/3 + 5/9 - 5/27 + \cdots$

6. Decide if each of the following series converges or diverges. If a series converges, find upper and lower bounds for its value.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[3]{k+1}}$

(b) $\sum_{k=1}^{\infty} \frac{(2k)!}{3^k (k!)^2}$

(c) $\sum_{k=1}^{\infty} \left(\frac{1}{100} + \frac{1}{k^5} \right)$

(d) $\sum_{k=1}^{\infty} \frac{\sqrt{9k^8 + 5k^6}}{12k^5 + 3}$

(e) $\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$

7. Does the first series from the previous problem converge absolutely or conditionally?

8. Compute the radius and interval (including endpoints) of convergence for $\displaystyle\sum_{k=1}^{\infty} \frac{(x+3)^k}{k \cdot 5^k}$.

9. Evaluate the following exactly.

(a) $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \cdots$

(b) $1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \cdots$

10. Using summation notation, write the series equal to $\int_0^1 e^{-x^2}\,dx$ and show that it converges.