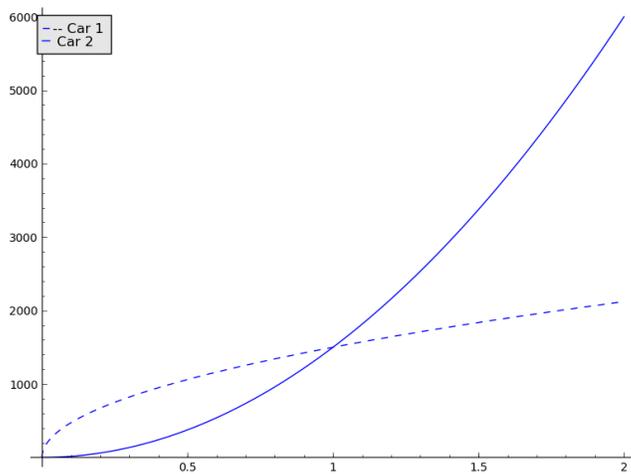


Name: _____

Final Exam

Show all your work to receive full credit for a problem. There are a total of 100 points on this test. Good luck!

1. (4 pts. each) Two cars start from rest at a traffic light and accelerate for several minutes. The figure below shows their velocities (in feet per minute) as a function of time (in minutes).



- (a) Which car is accelerating faster after one minute?
- (b) Which car is ahead after one minute?
- (c) Which car is ahead after two minutes?

2. (4 pts. each) Let $f(x) = 2x^3 - 13x^2 + 22x - 8$.

(a) Explain why the function f must have a root in the interval $(0, 1)$.

(b) Explain why the function f must have a root in the interval $(1, 3)$.

(c) Using parts (a) and (b), explain why there must be a stationary point in the interval $[0, 3]$.

(d) Explain why f must attain a maximum value on the interval $[0, 3]$, and find this value.

3. (5 pts each) For each of the following problems, find the equation of the line tangent to the graph at the given point.

(a) $y \sin 2x = x \cos 2y$ at the point $(\pi/2, \pi/4)$.

(b) $y = \arctan(x^2)$ at the point $(1, \pi/4)$.

(c) $y = \int_0^x \frac{3t^2}{1+t^3} dt$ at the point $(1, \ln 2)$.

4. (4 pts each) Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{\ln \sqrt{x}}{x^2}$

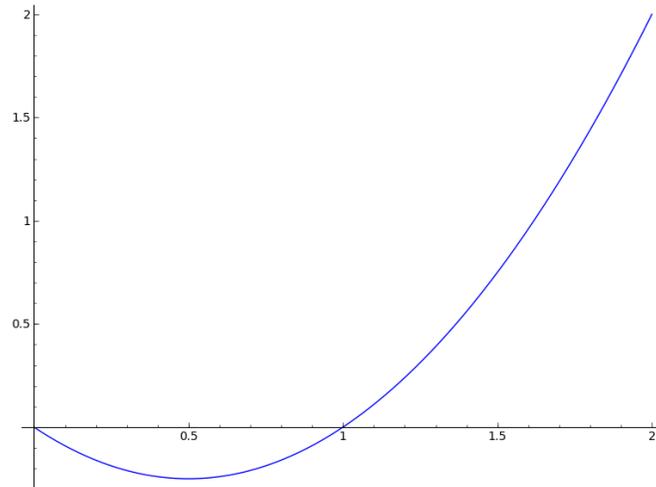
(b) $\lim_{x \rightarrow 0} \frac{\sec x}{1 - \sin x}$

5. (5 pts. each) Evaluate the following integrals:

(a) $\int_1^2 (8x^3 + 3x^2) dx$

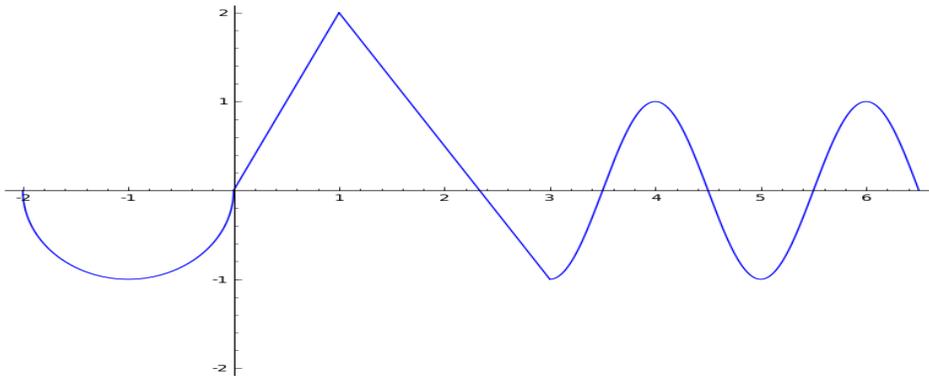
(b) $\int_0^1 \sin(3\pi x) dx$

6. (4 pts each) Consider the function $h(x) = x^2 - x$ on the interval $[0, 2]$. Below is the graph of h .



- (a) Partition the interval into four equal subintervals and use left approximating sums to compute an estimate of $\int_0^2 h(x)dx$. Make sure you draw the rectangles in the picture above.
- (b) Use the Fundamental Theorem of Calculus to calculate the exact value of the integral $\int_0^2 h(x)dx$.
- (c) How do the quantities in (a) and (b) compare to each other? Explain the limit definition of the integral using this example.

7. (3 pts each) Let $G(x) = \int_0^x g(t)dt$ where $g(t)$ is the function shown in the figure. Answer the following questions about G . (You may use estimates if you're not sure of the exact values.)



- (a) What is $G(-2)$? $G(1)$? $G(3)$?
- (b) What is $G'(1)$? What is $G''(1)$?
- (c) Where are the stationary points of G ?
- (d) On what intervals is G increasing? On what intervals is G decreasing?
- (e) What are the inflection points of G ?
- (f) On what intervals is G concave up? On what intervals is G concave down?

8. (9 pts) A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/sec. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?