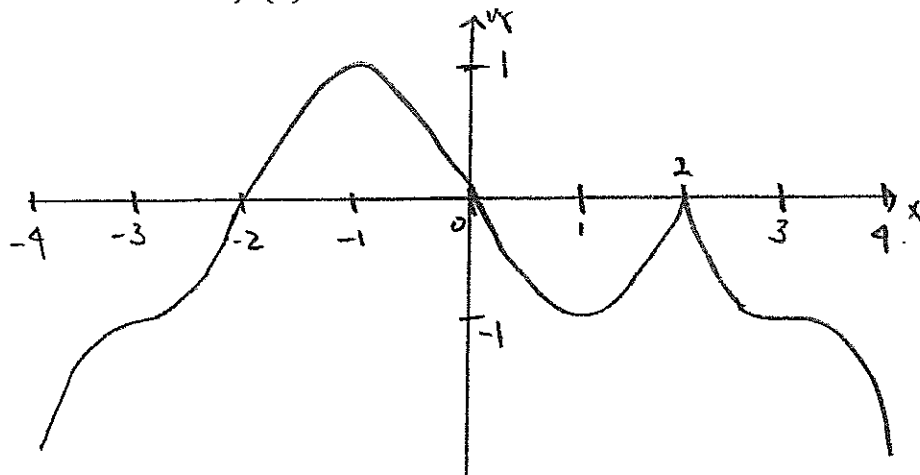


Math 105 A/B
Calculus 1
Prof. Balcomb
Final Exam
December 9, 2014

Instructions: Read each question carefully. Explain ALL your work and give reasons to support your answers.

1. The function f is **completely** defined by the graph below.

- What is the domain of f ?
- For which x -value(s) if $f'(x) = 0$?
- For which x -value(s) is $f'(x)$ undefined?
- On which interval(s) is $f'(x)$ positive?
- On which interval(s) is $f''(x) < 0$?
- Estimate the value of $f'(0)$.



2. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = 3x + 1$.

3. Find the derivative of the following functions:

a) $f(x) = (4x^3 - 2x)^4$

b) $f(x) = \cos(x) \ln(x)$

c) $f(x) = \arctan(4x)$

4. Find the equation of the tangent line to the graph of $x^3 + y^2 = 10$ at the point $(x,y) = (1,3)$.

5. Let V be the volume of a right cylinder having height h and radius r , and assume that h and r vary with time. At a certain instant, the height is 6 in. and increasing at 1 in/s, while the radius is 10 in. and decreasing at 1 in/s. How fast is the volume changing at that instant?

Note: the volume of a right cylinder is $V = \pi r^2 h$.

6. Suppose that a ball is thrown vertically upward and the height in feet of the ball t seconds after its release is modeled by the function

$$s(t) = -16t^2 + 29t + 6, \quad 0 \leq t \leq 2.$$

- a) Find the average rate of change of the height from $t = 0$ to $t = 1$.

- b) Find the instantaneous rate of change of the height at $t = \frac{1}{2}$.

7. An open box is to be made from a 3-ft by 8-ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the box that produces a maximum volume.

8. Let $f(x) = x^4 - 4x^3$.

a) Using calculus, find the critical numbers for f .

b) Using calculus, classify the critical number(s) as a relative maximum, relative minimum, or neither.

c) Using calculus, find the x-value of any points of inflection.

9. Given: $f(x) = x^3$

a) Estimate $\int_1^2 x^3 dx$ using L_4 (explicitly show the numbers in the sum L_4 and compute their sum).

b) Find the exact value of $\int_1^2 x^3 dx$ using the FTC.

c) What is the average value of $f(x) = x^3$ on the interval $[0,2]$?

10. Consider the initial value problem (IVP) $\frac{dy}{dx} = 0.4y$ where $y(0) = 2$. For what values of A and B does $y = Ae^{Bx}$ satisfy the IVP as given?

11. Evaluate the following:

a) $\int_0^1 (x^2 - 4x + 7) dx$

b) $\int_1^8 (5x^{2/3} - \sqrt{x}) dx$

c) $\int_1^2 \frac{1}{x^6} dx$

d) $\int_1^2 (4^x + \frac{1}{x}) dx$

12. Find the following limits:

a) $\lim_{x \rightarrow 0} \frac{\arcsin(2x)}{x}$

b) $\lim_{x \rightarrow 0^+} \frac{1 - \ln(x)}{e^{1/x}}$

13. Find the limit: $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\sqrt{9 - \left(i \frac{3}{n}\right)^2} \left(\frac{3}{n}\right) \right)$

