Math 106: Review for Final Exam, Part I

1. Find the following. [See Review for Exam II for integration tips and strategies.]

   (a) $\int 12x^2 \cos(x^3)\, dx$

   (b) $\int_0^\infty xe^{-3x}\, dx$

   (c) $\int_0^6 \dfrac{dx}{(x-4)^2}$

   (d) $\int \dfrac{3x^2+2x-5}{(x^2+1)(x-4)}\, dx$

   (e) $\int_0^{\pi/3} \tan^3 x \sec^5 x\, dx$

(f) $\int \sqrt{25 - x^2}\, dx$

2. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to $\int_{-2}^{0} f(x)\, dx$ given the data in the table below.

x	-2	-1.5	-1	-0.5	0
$f(x)$	2	3	6	10	11

3. If you use numerical integration to estimate $\int_a^b \ln x\, dx$ (where a and b are positive), how would the following be ordered from least to greatest? L_{100}, R_{100}, M_{100}, T_{100}, $\int_a^b \ln x\, dx$.

4. Find bounds for each of the following errors if $I = \int_0^2 e^{-5x}\,dx$.

   (a) $|I - R_{100}|$

   (b) $|I - T_{100}|$

   (c) $|I - M_{100}|$

5. If $I = \int_0^2 e^{-5x}\,dx$, how many subdivisions are required to obtain a midpoint sum approximation with error of at most 1/1,000,000?

6. Write an integral equal to the area between $y = 2x + 3$ and $y = x^2 + 7x - 3$.

7. Compute the arc length of $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1/2$.

8. Consider the region bounded by $y = 0$, $x = 2$, and $y = x^2$. Write an integral equal to the volume of the object created when the region is revolved about

   (a) the x-axis

   (b) the line $x = 5$

9. The probability density function (pdf) of the weights of newborn toads in a certain pond is given by $f(x) = \dfrac{k}{(x+1)^4}$, where x is the weight (in ounces). Note that the domain is $x \geq 0$ since no toad can have a negative weight.

   (a) What must be the value of k?

   (b) What fraction of the newborn toads weigh more than one ounce?

10. Find the solution to $\dfrac{dy}{dx} = \dfrac{\cos x}{y^2}$ that passes through $(0, 2)$.