

Math 106: Review for Final Exam, Part I

1. Find the following. [See Review for Exam II for integration tips and strategies.]

(a) $\int 12x^2 \cos(x^3) dx$

(b) $\int_0^{\infty} xe^{-3x} dx$

(c) $\int_0^6 \frac{dx}{(x-4)^2}$

(d) $\int \frac{3x^2 + 2x - 5}{(x^2 + 1)(x - 4)} dx$

(e) $\int_0^{\pi/3} \tan^3 x \sec^5 x dx$

(f) $\int \sqrt{25 - x^2} dx$

2. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to $\int_{-2}^0 f(x) dx$ given the data in the table below.

x	-2	-1.5	-1	-0.5	0
$f(x)$	2	3	6	10	11

3. If you use numerical integration to estimate $\int_a^b \ln x dx$ (where a and b are positive), how would the following be ordered from least to greatest? L_{100} , R_{100} , M_{100} , T_{100} , $\int_a^b \ln x dx$.

4. Find bounds for each of the following errors if $I = \int_0^2 e^{-5x} dx$.

(a) $|I - R_{100}|$

(b) $|I - T_{100}|$

(c) $|I - M_{100}|$

5. Write an integral equal to the area between $y = 2x + 3$ and $y = x^2 + 7x - 3$.

6. Compute the arc length of $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1/2$.

7. Consider the region bounded by $y = 0$, $x = 2$, and $y = x^2$. Write an integral equal to the volume of the object created when the region is revolved about

(a) the x -axis

(b) the line $x = 5$

8. A spherical tank of radius 8 feet is buried 5 feet below ground and filled to a height of 11 feet with gasoline (42 pounds per cubic foot). Write an integral equal to the work done in pumping all the gasoline to ground level.

9. The probability density function (pdf) of the weights of newborn toads in a certain pond is given by $f(x) = \frac{k}{(x+1)^4}$, where x is the weight (in ounces). Note that the domain is $x \geq 0$ since no toad can have a negative weight.

(a) What must be the value of k ?

(b) What fraction of the newborn toads weigh more than one ounce?

10. Find the solution to $\frac{dy}{dx} = \frac{\cos x}{y^2}$ that passes through $(0, 2)$.