

Math 105: Review for Final Exam, Part I

1. Consider the function $f(x) = \frac{3}{5 - 2x}$.

(a) Is this function continuous on the interval $(-\infty, \infty)$? Explain.

(b) Compute the average rate of change of f on $[2, 2.01]$.

(c) Using the limit definition of the derivative, compute $f'(x)$.

(d) Find the equation of the tangent line to f at $x = 2$.

2. Given that $f(0) = 2$, $g(0) = 3$, $f'(0) = 5$, $g'(0) = 7$, and $f'(3) = \pi$ compute the following.

(a) $h'(0)$ if $h(z) = f(z)g(z)$

(b) $j'(0)$ if $j(z) = \frac{f(z)}{g(z)}$

(c) $k'(0)$ if $k(z) = f(g(z))$

3. (a) Find $\frac{dy}{dt}$ if $y = t^5 + 5^t + e^5 + \frac{t}{5} + \frac{5}{t} + \frac{5}{\sqrt[5]{t}} + \ln(5t) + \arctan(5t) + \ln(5) + \sin 5$.

(b) Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x} \cos(7x^3)$.

(c) Find $\frac{dy}{dz}$ if $y = \frac{e^z + e^\pi}{\tan 4 - 7z}$.

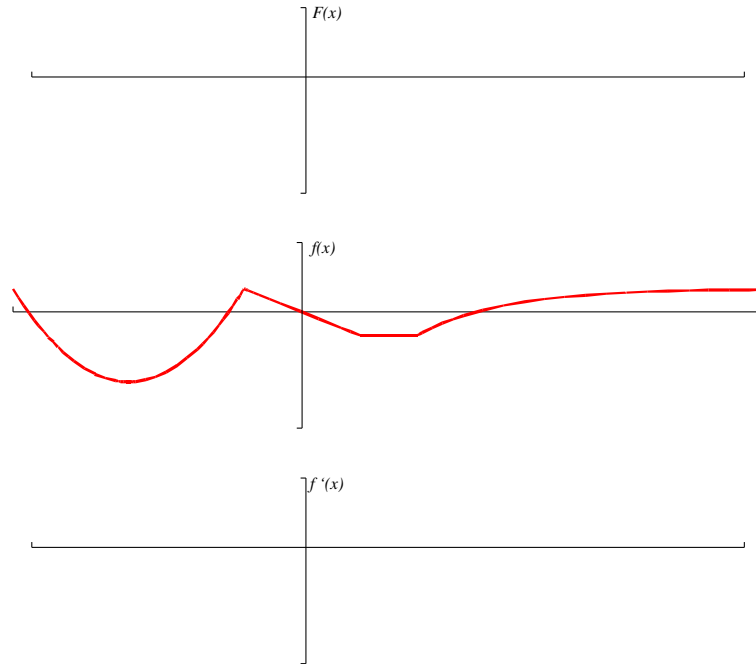
(d) Find $\frac{dy}{dr}$ if $y = \tan(e^{r^2} \arcsin(5r))$.

(e) Find $\frac{dy}{dx}$ if $y^3 + yx^2 + x^2 = 3y^2$.

(f) Find $\frac{dy}{dx}$ if $y = (1 + x^6)^{8x}$.

Hint: use logarithmic differentiation.

4. Given the graph of f , sketch a graph of f' and a graph of F , an antiderivative of f such that $F(0) = -1$.

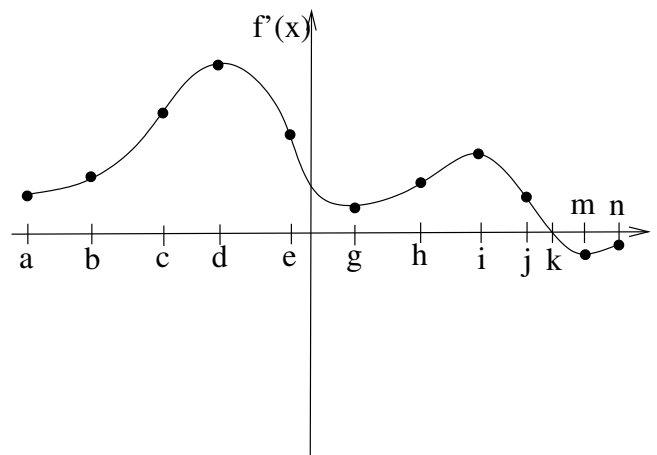


5. Shown below is a graph of f' on its entire domain. The graph is NOT f .

At which x -value(s) (if any)

- (a) does f have a stationary point?
- (b) does f have a local max?
- (c) does f have a local min?
- (d) does f' have a stationary point?
- (e) does f' have a local max?
- (f) does f' have a local min?
- (g) is f greatest?
- (h) is f least?
- (i) is f' greatest?
- (j) is f' least?
- (k) is f'' greatest?
- (l) is f'' least?

- (c) f' increasing?
- (d) f' decreasing?
- (e) f concave up?
- (f) f concave down?



On what interval(s) is

- (a) f increasing?
- (b) f decreasing?

6. Solve the IVP $y' = e^x - \sin x + 5$ given that $y(0) = 3$.

7. Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{x^2}{\ln x}$

(b) $\lim_{z \rightarrow 0} \frac{\sin(5z) - 5z}{z^3}$

(c) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\cos x}$

(d) $\lim_{r \rightarrow 2} \frac{r^3 - 8}{r - 2}$

8. Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.

(a) Find the x - and y -coordinates of any and all critical points and classify each as a local maximum, local minimum, or neither.

(b) Find the x - and y -coordinates of any and all global extrema and classify each as a global maximum or global minimum.

(c) Find the x -coordinate(s) of any and all inflection points.