Math 105 Quiz 8
§5.6-5.7, 5.3
Name:
Show all work for credit.
Toolbox:

$$\sum_{i=1}^{n} 1 = n, \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. Using infinite Riemann Sums, $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$, determine the following area.

$$\int_0^2 x^3 + 2x^2 - 1$$

$$\Delta x = \frac{2}{n}, x_i^* = \frac{2i}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}\right)^3 + 2\left(\frac{2i}{n}\right)^2 - 1\right)\frac{2}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i^3}{n^3} + \frac{8i^2}{n^2} - 1\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{16}{n^4} \sum_{i=1}^{n} i^3 + \frac{16}{n^3} \sum_{i=1}^{n} i^2 - \frac{2}{n} \sum_{i=1}^{n} 1$$

$$= \lim_{n\to\infty} \frac{16}{n^4} \left(\frac{n(n+1)}{2}\right)^2 + \frac{16}{n^3} \frac{n(n+1)(2n+1)}{6} - \frac{2}{n} n = 4 + \frac{16}{3} - 2 = \frac{22}{3}$$

Verify your answer using the FTC.

$$\int_0^2 x^3 + 2x^2 - 1 = \left.\frac{x^4}{4} + \frac{2x^3}{3}\right|_0^2 = 4 + \frac{16}{3} - 2 = \frac{22}{3}$$