

MATH 205A,B - LINEAR ALGEBRA
FALL 2015

QUIZ 9

NAME:

Section:(Circle one) A(8 : 00) B(9 : 30)

Show **ALL** your work **CAREFULLY**.

Let

$$\vec{u}_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ -1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} -3 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \end{bmatrix}.$$

(a) Show that the set $S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthogonal set.

Note that

$$\vec{u}_1 \cdot \vec{u}_2 = (4)(-3) + (3)(4) + (2)(1) + (-1)(2) = 0$$

$$\vec{u}_2 \cdot \vec{u}_3 = (-3)(1) + (4)(-2) + (1)(3) + (2)(4) = 0$$

$$\vec{u}_1 \cdot \vec{u}_3 = (4)(1) + (3)(-2) + (2)(3) + (-1)(4) = 0.$$

It follows that these vectors are pairwise orthogonal so that S is an orthogonal set.

(b) Let $\vec{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. Find the projection of \vec{y} onto $W = \text{Span}\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$.

Sine S is an orthogonal set,

$$\begin{aligned} \text{proj}_W \vec{y} &= \text{proj}_{\vec{u}_1} \vec{y} + \text{proj}_{\vec{u}_2} \vec{y} + \text{proj}_{\vec{u}_3} \vec{y} \\ &= \frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} \vec{u}_1 + \frac{\vec{y} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} \vec{u}_2 + \frac{\vec{y} \cdot \vec{u}_3}{\vec{u}_3 \cdot \vec{u}_3} \vec{u}_3 \\ &= \frac{8}{30} \vec{u}_1 + \frac{4}{30} \vec{u}_2 + \frac{6}{30} \vec{u}_3 = \frac{1}{15} \begin{bmatrix} 13 \\ 14 \\ 19 \\ 12 \end{bmatrix}. \end{aligned}$$

(c) Let \vec{y} and W be as in part (b). Find the point in W closest to \vec{y} .

The closest point in W is $\text{proj}_W \vec{y} = \frac{1}{15} \begin{bmatrix} 13 \\ 14 \\ 19 \\ 12 \end{bmatrix}$.