Math 105 Quiz 7
§4.8, 4.9, 5.1
Name:
Show all work for credit.

1. State the Mean Value Theorem.

   If f is a continuous function on the closed, bounded interval $[a, b]$ and differentiable on (a, b) then there exists a c in the interval $[a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

2. Verify the hypothesis of the Intermediate Value Theorem in the interval [0,5] for the function $f(x) = x^2 + x - 1$ and find the value of c guaranteed by theorem when $f(c) = 11$.

   $f(x) = x^2 + x - 1$ is continuous on the closed, bounded interval [0,5]. $f(0) = -1$ and $f(5) = 29$. Since 11 is between -1 and 29, there exists a c between 0 and 5 such that $f(c) = 11$.

$$f(c) = c^2 + c - 1 = 11 \to c^2 + c - 12 = 0 \to (c-3)(c+4) = 0 \to c = 3.$$

3. Suppose $\int_{-2}^{2} f(x) = 7$, $\int_{2}^{8} f(x) = 4$, and $g(x)$ is an odd function. Find the following:

   (a) $\int_{-2}^{8} f(x)dx = 7 + 4 = 11$

   (b) $\int_{-2}^{2} [f(x) + g(x)]dx = 7 + 0 = 7$

   (c) $\int_{0}^{4} (x - 1)dx = \frac{-1}{2} + \frac{9}{2} = 4$