

Math 205 Quiz 9

Name:

1. What is the 1-norm, 2-norm, and ∞ -norm for $\vec{t} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$?

$$\|\vec{t}\|_1 = 5, \|\vec{t}\|_2 = \sqrt{11}, \|\vec{t}\|_\infty = 3$$

2. Find the distance between $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 4 \\ 1/2 \\ 0 \end{bmatrix}$.

$$\|\vec{x} - \vec{y}\| = \left\| \begin{bmatrix} -3 \\ -3/2 \\ 2 \end{bmatrix} \right\| = \sqrt{9 + \frac{9}{4} + 4} = \sqrt{\frac{61}{4}} = \sqrt{15.25} \approx 3.905$$

3. Find the angle between $\vec{x} = \begin{bmatrix} 1 \\ \sqrt{2} \\ -1 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

$$\cos(\theta) = \frac{\vec{x} \cdot \vec{y}}{\|\vec{x}\| \|\vec{y}\|} = \frac{\sqrt{2}}{2}, \theta = \frac{\pi}{4}$$

4. Find the unit vector associated with $\vec{v} = \begin{bmatrix} 2 \\ \sqrt{3} \\ 1 \end{bmatrix}$.

$$\|\vec{v}\| = \sqrt{4 + 3 + 1} = \sqrt{8} = 2\sqrt{2}$$

The unit vector will be

$$\frac{\vec{v}}{2\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{bmatrix}.$$

5. Is the following set an orthogonal set?

$$\left\{ \begin{bmatrix} 1 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ \sqrt{2} \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ \sqrt{2} \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} -1 \\ \sqrt{2} \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0$$

No, it is not an orthogonal set.