Math 105 Name SOLUTIONS

Quiz 6: In-Class and Open Notes

NO CALCULATORS ALLOWED

11/16/12

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. *Don't forget, you are not allowed to use a calculator for this quiz!*

1. **You must work on this quiz with at least one other person but no more than 3 other people.** Please list the names of all students with whom you collaborated on this in-class, open notes quiz.

2. (5 pts) Consider $f(x) = 2x^3 - x - 2$.

   (a) Find two values, a and b, such that $f(a) < 0$ and $f(b) > 0$.

   let $a=1$ then $f(a)=f(1)=2-1-2=-1$

   let $b=2$ then $f(b)=f(2)=16-2-2=12$

   (b) State the first hypothesis of the *Intermediate Value Theorem*. In other words, what nice property do we require of f on $[a, b]$?

   f is continuous on $[1,2]$ ← we know this since f is a polynomial and polynomials are continuous for all values of x

   (c) Let $y = 0$, notice that $f(a) < y < f(b)$. What does the Intermediate Value Theorem allow you to conclude about f on $[a, b]$?

   Since f is cont on $[1,2]$ and $f(1)<0<f(2)$, then by the IVT there exists c between 1 & 2 so that $f(c)=0$.
   In other words, $f(x)=2x^3-x-2$ has a root on $[1,2]$.

3. (5 pts) Consider $f(x) = \sqrt{x}$.

   (a) State the hypothesis of the *Extreme Value Theorem*.

   f must be continuous on a closed & bounded interval $[a,b]$.

   (b) What does the *Extreme Value Theorem* say about f on the interval $[0, 1]$?

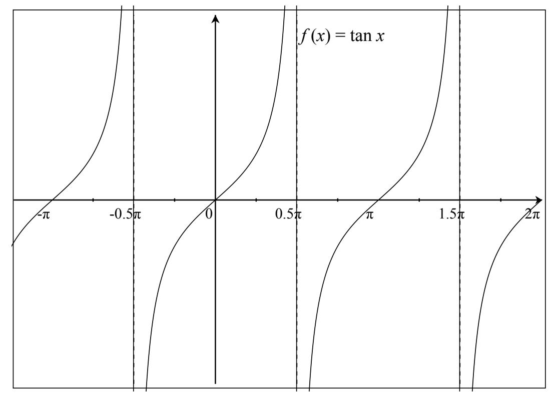
   Since $f(x)=\sqrt{x}$ is continuous on $[0,1]$ then f achieves both a (global) max and a (global) min on $[0,1]$.

   (c) Does the *Intermediate Value Theorem* hold for $f(x) = \sqrt{x}$ on $[0, 4]$ and $y = 3$? Explain your answer.

   Note: $f(0)=0$ & $f(4)=2$, but $y=3$ is NOT between $f(0)$ & $f(4)$.
   Therefore, we haven't satisfied the 2^{nd} hypothesis of the IVT.
   In other words, the IVT doesn't apply so we can't use it to make conclusions about this case.

4. (5 pts) Consider $f(x) = \tan x$.

(a) State all hypotheses of the *Mean Value Theorem.*



1. f must be continuous on $[a, b]$
2. f must be differentiable on (a, b)

(b) Are the hypotheses listed in (a) satisfied by the function $f(x) = \tan x$ on the interval of x-values $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$? *Be specific about how all hypotheses are satisfied, or about which hypotheses fail.*

The graph of $f(x) = \tan x$ has an asymptote at $x = \frac{\pi}{2}$

therefore, f is neither continuous on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$,

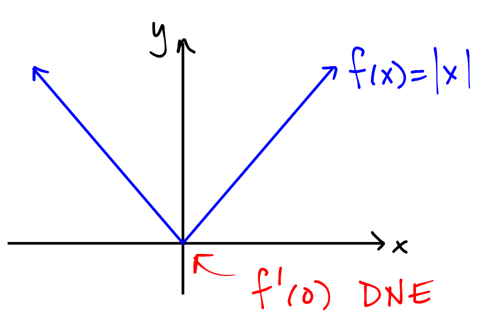
nor differentiable on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

In other words, the MVT does not apply in this case.

5. (5 pts) Each of the following is **FALSE**. Give an example to show why the statement is false.

(a) If f is continuous, then f is differentiable.

$f(x) = |x|$ is continuous on $(-\infty, \infty)$, but is not differentiable at $x = 0$.



(b) If $F'(x) = G'(x)$ then $F(x) = G(x)$ for all values of x.

let $F(x) = x^2$ and $G(x) = x^2 + 2$

clearly $F(x) \neq G(x)$ for any value of x

while $F'(x) = 2x = G'(x)$ for all values of x.