Math 105 C Quiz 07 ver01 page 1 11/15/2013 **Name** suggested solution

1. Consider the function $f(x) = \begin{cases} 3x^2 & \text{if } x \le -1 \\ A + Bx^3 & \text{if } x > -1 \end{cases}$

1A) Find the conditions on A and B that make this function continuous at $x = -1$. Show all your work.

we need $\lim_{x\to -1^-} f(x)$ to equal $\lim_{x\to -1^+} f(x)$.

Now, $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} 3x^2 = 3(-1)^2 = 3$

and $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} A + Bx^3 = A + B(-1)^3 = A - B$.

So the one-sided limits are equal iff $\boxed{3 = A - B}$

1B) Find conditions on A and B that make this function differentiable at $x = -1$. Show all your work.

Now we need $\lim_{x\to -1^-} (3x^2)' = \lim_{x\to -1^+} (A+Bx^3)'$

or, $\lim_{x\to -1^-} (6x) = \lim_{x\to -1^+} (0 + 3Bx^2)$

or, $-6 = 3B(-1)^2$

or $-6 = 3B$ $\therefore \boxed{B = -2}$

from 1A, we (still) need $3 = A - B$

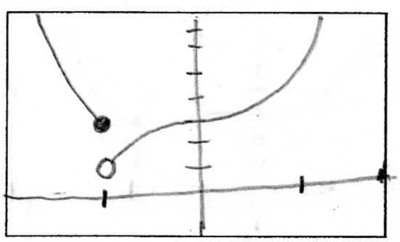
$\therefore$ $3 = A - (-2)$

$3 = A + 2$

$\boxed{A = 1}$

1C) Let $A = 3$ and $B = 2$. Do these values satisfy the conditions in 1A? $\boxed{\text{NO}}$ Set your calculator window to $[-2, 2] \times [-1, 8]$ and use your calculator to help draw the correct graph of f (still using $A = 3$ and $B = 2$). Draw a careful facsimile of the correct graph here (including the axes and the "tick marks": set `Xscl=1` and `Yscl=1` in the Window menu).

because of $A=3$ & $B=2$ we get $A - B = 3 - 2 = 1$ but continuity requires $A - B = 3$.



the calculator HELPS but doesn't give the DETAILS that you know about:

2. The *Intermediate Value Theorem* assumes that f is a continuous function on an interval $[a, b]$, and that d is chosen between $f(a)$ and $f(b)$. What is the conclusion of the theorem?

that there must be some c in $[a,b]$ for which $f(c) = d$

3. The *Extreme Value Theorem* assumes that f is a continuous function on an interval $[a, b]$, and it says that f "assumes" (or "achieves" or "attains") its maximum value on $[a, b]$. Explain what this means.

It means that for some $x_0 \in [a,b]$, $f(x_0) \geq f(x)$ for all (other) x's in $[a,b]$

4. How is the IVT used to prove there must be a root of $x^5 - 4x^3 + 1$ somewhere between $x = 0$ and $x = 1$?

Call this polynomial "$p(x)$". Now, $p(0) = 1$ and $p(1) = -2$

choose $d = 0$; because $\left\{ \begin{array}{c} -2 \le 0 \le 1 \\ \text{OR} \\ p(1) \le d \le p(0) \end{array} \right\}$, there must be a c between 0 and 1 for which $p(c) = d$, ie. $p(c) = 0$