

Math 205 Quiz 8

Name: *Key*

1. Is $\vec{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} -3 & 1 \\ -3 & 5 \end{bmatrix}$? Explain.

$$\begin{bmatrix} -3 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 17 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 17 \end{bmatrix}$ is not a multiple of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$. $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ is not an eigenvector.

2. Find the characteristic equation for A and determine the eigenvalues of A .

$$\begin{vmatrix} -3-\lambda & 1 \\ -3 & 5-\lambda \end{vmatrix} = (-3-\lambda)(5-\lambda) + 3 = \boxed{\lambda^2 - 2\lambda - 12 = 0}$$

$$\lambda = \frac{2 \pm \sqrt{4 + 48}}{2} = \underline{\underline{1 \pm \sqrt{13}}}$$

3. Is $\lambda = 1$ an eigenvalue of $B = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$? Explain.

$$B - I = \begin{bmatrix} 2 & 2 \\ 3 & 7 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ which has a trivial nullspace. } \lambda = 1 \text{ is not an eigenvalue of } B.$$

4. The eigenvalues of C are $\lambda = 1, 5$.

$$C = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$

If possible, determine P and D such that $C = PDP^{-1}$.

$$\lambda = 1 \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 1 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Basis} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\lambda = 5 \Rightarrow \begin{bmatrix} -3 & 2 & -1 \\ 1 & -2 & -1 \\ -1 & -2 & -3 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Basis} = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

$$P = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$