

Name: _____

Exam 2 – Math 105

Show all your work to receive full credit for a problem. There are a total of 72 points on this test. Good luck!

1. (4 points each.) Calculate each of the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{x - 1}{2 - 3x^2}$

(b) $\lim_{x \rightarrow 3} \frac{2^x - 8}{3 - x}$

(c) $\lim_{x \rightarrow 1} \frac{\sin(\ln x)}{x - 1}$

2. (4 points each.) Suppose g , h , and j are differentiable functions with the values for the function and derivative given in the following table:

x	$g(x)$	$h(x)$	$j(x)$	$g'(x)$	$h'(x)$	$j'(x)$
-2	1	2	3	2	-3	0
-1	3	0	1	-1	-2	-2
0	2	3	0	-2	3	-2
1	0	-1	-2	-2	-2	-1
2	-2	-2	-3	-1	0	2
3	-3	0	1	0	1	2

(a) If $f(x) = g(x)(h(x) + j(x))$, find $f'(2)$.

(b) If $f(x) = \frac{3h(x)}{g(x) + j(x)}$, find $f'(0)$.

(c) If $f(x) = g(x^3 - 6)$, find $f'(2)$.

(d) If $f(x) = h(g(j(x)))$, find $f'(1)$.

3. (6 points each.) Find all the critical points for each of the following functions, if there are any.

(a) $f(x) = \arctan(x^2)$.

(b) $f(x) = \cos^2(x)$ in the interval $[0, 2\pi]$.

4. (2 points each.) For each of the following, give an example of a real-world problem that can be modeled using these functions. Explain the problem as carefully as possible in the amount of space given.

(a) The exponential function.

(b) The sine and cosine functions.

5. (8 points.) Use logarithmic differentiation to find the derivative of the function $f(x) = x^{\sin(3x)}$.

6. (8 points.) Find the equation of the tangent line to the graph of

$$\ln(x^2 - 3y) = x - y - 1$$

at the point $(2, 1)$.

7. (12 points) A rectangular storage container with an open top is to have a volume of 22 cubic meters. The length of its base is twice the width. Material for the base costs 10 dollars per square meter. Material for the sides costs 5 dollars per square meter. Find the cost of materials for the cheapest such container. (Round to the nearest penny and include monetary units. For example, if your answer is 1.095, write \$1.10 including the dollar sign and second decimal place.)