Calculus I - Section A  Chapters \{2,3,4\} Proficiency Assessment November 13, 2015

Name: ________________________________________________________________

1. Do not open this booklet until you are told to do so.

2. Try not to separate the pages. If they do become separated, write your names on every page and point this out to your proctor when you hand it in.

3. Show an appropriate amount of work (including appropriate explanation) for each problem and not just the final answer. Include units in your answer where that is appropriate.

4. You may use any calculator functionally equivalent to a TI-83/TI-83+ or TI-84/TI-84+. Use of calculators with more functionality than these is not allowed.

5. Turn off all cell phones and pagers, and remove all headphones.

Proficiency Level on Chapter 2:__________

Proficiency Level on Chapter 3:__________

Proficiency Level on Chapter 4:__________
(I) Use the graphs of \( f \) and \( g \) shown below to evaluate the limit or explain why the limit doesn’t exist.

(a) \( \lim_{x \to -1} (f(x) + g(x)) = \)

(b) \( \lim_{x \to 1^+} g(x) = \)

(c) \( \lim_{x \to 1^-} g(x) = \)

(d) \( \lim_{x \to 1} (f(x) \cdot g(x)) = \)

(e) \( \lim_{x \to 2} \left( \frac{f(x) + 2}{f(x)(g(x) + 2)} \right) = \)

(II) If \( 2x \leq g(x) \leq x^4 - x^2 + 2 \) for all \( x \), evaluate \( \lim_{x \to 1} g(x) \)
Chapter 2 Proficiency Test - Problem 2

(I) Let \( f(x) = 3x^2 - 8x - 2 \). Evaluate

\[
\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}
\]

(II) If the velocity of a particle at any time \( t \) is \( v(t) = 15t^4 - 75t^2 + 60 \), find the position \( x(t) \) of the function given that \( x(1) = 38 \). Recall that \( v(t) = x'(t) \).

(II) Find \( c \) so that the line \( y = 4x + 3 \) is tangent to the curve \( y = x^2 + c \).
Chapter 3 Proficiency Test - Problem 1

(I) Find the solution of the second-order IVP

\[ y'' = -4y, \quad y'(0) = 2, \quad y(0) = 6 \]

(II) A mold grows at a rate proportional to the amount present. Initially, its weight is 2 g; after 2 days, it weighs 5 g. How much does it weigh after 8 days?
(I) If \( xy + y^2 = 1 \), find the value of \( y'' \) at the point \((0, -1)\).

(II) Find the derivative of the function

\[
f(x) = \frac{x^{2/3} + \log_5(x)}{e^{x^2} \cos(4x)}
\]
Chapter 4 Proficiency Test - Problem 1

Rewrite

\[ \tan(\arcsin(2x)) \]

as an algebraic expression. Be sure to draw the right angle triangle and label all the sides correctly.
(I) Find \( \lim_{\theta \to 5} \frac{\theta - 5}{\theta^2 - 25} \)

(II) Suppose that \( f \) and \( g \) are differentiable functions and that \( f(0) = 5, \ g(0) = 7 \). Find
\[
\lim_{x \to 0} \frac{xf(x)}{(e^x - 1)g(x)}
\]
(III) Evaluate \( \lim_{x \to \infty} x (\pi - 2 \arctan x) \)

(IV) Evaluate \( \lim_{x \to \infty} (\sqrt{x^4 + 5x^2} - x^2) \)
Chapter 4 Proficiency Test - Problem 3

A farmer wants to fence a rectangular area as inexpensively as possible. Assuming that fencing materials cost $1 per foot. Suppose that $40 is available for the project. How much area can be enclosed?