1. Consider the function $f(x, y) = 2x^3 - y^3 + 3x^2 - 36x + 3y$.

1a) Find ∇f.

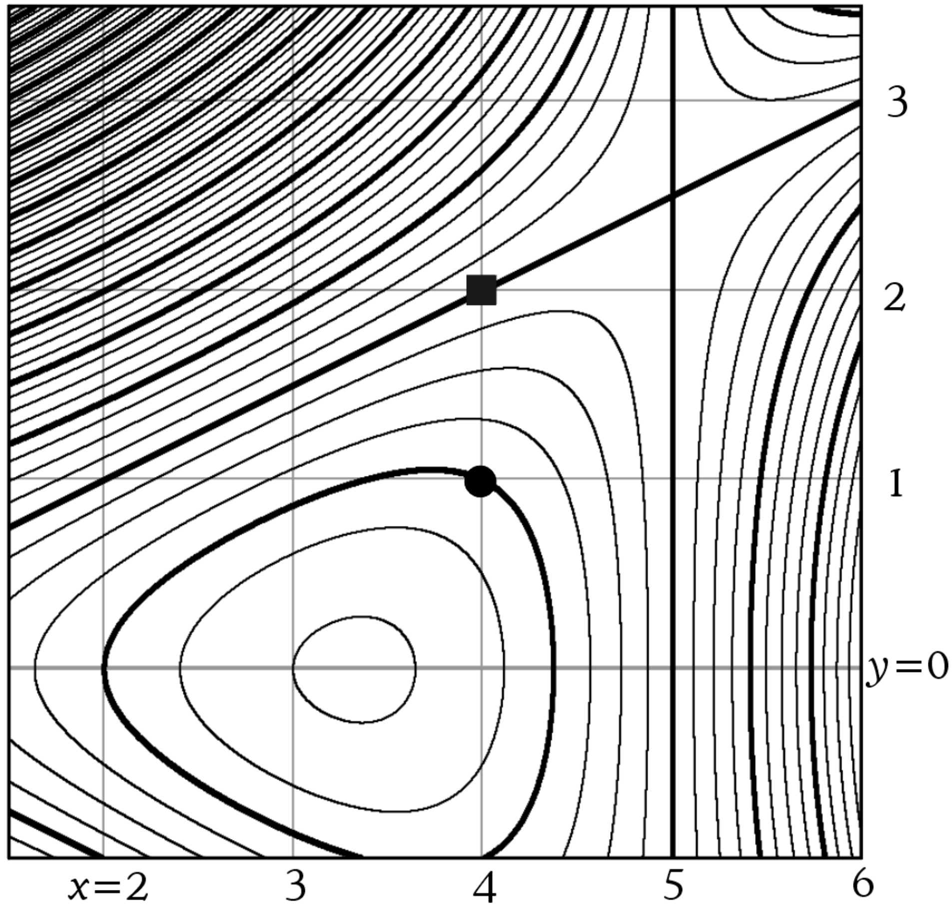
1b) Find all the critical points of f.

1c) Classify each of the critical points in problem (1b) as local max/mins/saddle points according to the second derivative test. If the test fails, explain why. ORGANIZE your answers neatly.

2. Let $f(x, y) = 5\,x^2 - x^3 - 20\,y^2 + 4\,xy^2$. The accompanying graph shows level curves at intervals of 3, with those at intervals of 12 drawn with heavy lines.

2A) Find ∇f.

2B) There's a dot at (4,1). In the form $y = mx + b$, find the equation of the line tangent to the level curve there. Show all your work. Then make an excellent sketch, with the right slope, of this line on your graph.



2C) Find a *unit* vector in the direction of the gradient at the point (4,1). Accurately sketch this vector on the graph, with its "tail" at (4,1).

2D) There appears to be a local maximum at some point $(p, 0)$ on the x-axis to the right of $(3, 0)$. Use ∇f to find the value of p exactly.

2E) What is the equation of the plane tangent to the graph of f at this local maximum (ie, at $(p, 0, f(p, 0))$? (this is an easy question: think about what the plane must look like at a maximum)

(this is a continuation of problem 2)

2F) At the point (4,2), marked with a black square (■), the level curve looks like a straight line. Find in the form $y = mx + b$ the equation of this level "curve" just by looking at the graph.

2G) What is the value of f on that level curve (ie, the line) in 2F?

2H) BONUS: show analytically that all the points on the line in 2F are indeed on that level curve.

2I) (not a bonus) Find the directional derivative in the direction of the vector $(4, 5)$ at the point $(4, 1)$.

2J) (not a bonus) Find the equation of the plane tangent to the graph of $f(x, y)$ at the point $(4, 1, f(4, 1))$

4A. Find the second order Taylor polynomial for $f(x, y) = y\sqrt{x}$ for the point $\mathbf{a} = (25, 3)$.

4B) Use that polynomial from 4A to approximate $3.02\sqrt{26}$ (that is, $f(26, 3.02)$). Compare the approximation to the actual (calculator) value by finding the differences.

5. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = (x^2yz^3,\ 5x + 4y + 3z,\ x^2 + y^2z^2)$ and find both the divergence and curl of this vector field.

5A: the divergence is:

5B the curl is:

6A. Suppose $\mathbf{g}(r, p, s)$ has two component functions g_1 and g_2. Suppose each of r, p and s is a function of x and y; let $\mathbf{f}(x, y)$ have r, p and s as its component functions. With this much information, find (the entries of) both $J\mathbf{g}$ and $J\mathbf{f}$. (for example, one of the entries in $J\mathbf{g}$ is $\partial g_2/\partial p$; one of the entries in $J\mathbf{f}$ is $\partial p/\partial y$).

6B. Let $\mathbf{u} = \mathbf{g} \circ \mathbf{f}$, then $\mathbf{u}$ is a function of x and y. Let its component functions be u_1, u_2, etc. Use the chain rule and the Jacobians in 6A to find an expression for $\partial u_2/\partial y$.