MATH 205A,B - LINEAR ALGEBRA
FALL 2015

QUIZ 8

NAME: **Section:**(Circle one) A(8 : 00) B(9 : 30)

Show ALL your work CAREFULLY.

Let

$$A = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues of A.

The eigenvalues of A are the solutions of the characteristic equation $\det(A - \lambda I) = 0$. Since

$$\det(A - \lambda I) = \det \begin{bmatrix} 6-\lambda & -3 \\ 2 & 1-\lambda \end{bmatrix} = \lambda^2 - 7\lambda + 12 = (\lambda - 4)(\lambda - 3),$$

it follows that the eigenvalues are 4 and 3.

(b) For each of the eigenvalue(s) found in (a), find the corresponding eigenspace.

For any eigenvalue λ, the corresponding eigenspace is the null space of the matrix $(A - \lambda I)$.

For $\lambda = 4$, consider

$$A - 4I = \begin{bmatrix} 2 & -3 \\ 2 & -3 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}.$$

It follows that the null space of $A - 4I$ is $\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \right\}$.

Similarly for $\lambda = 3$, consider

$$A - 3I = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}.$$

It follows that the null space of $A - 3I$ is $\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

(c) Is A diagonalizable? If so, find an invertible matrix P such that $P^{-1}AP$ is diagonal.

Since A has two distinct eigenvalues and thus two linearly independent eigenvectors, A is diagonalizable. The matrix $P = \begin{bmatrix} \frac{3}{2} & 1 \\ 1 & 1 \end{bmatrix}$ has the property that $A = PDP^{-1}$ where $D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$. It is straightforward to check that $AP = PD$.

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