Math 105a
Exam 2
11/11/11

Name _____

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

1. (21 pts) Find the derivative of each function below. Simplify your answers.

   (a) $f(x) = x^3 \cos(4x)$

   (b) $f(x) = \dfrac{2^x - e^x}{e^x}$

   (c) $f(x) = \ln\left(e^{\sin x + x^2}\right)$

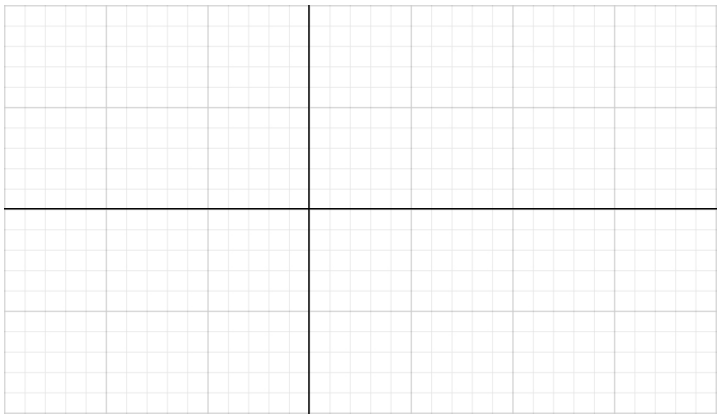
2. (15 pts) Consider the function f, defined for all real numbers, with first and second derivatives given below:

$$f'(x) = \frac{5(x-4)}{3\sqrt[3]{x}} \quad \text{and} \quad f''(x) = \frac{10(x+2)}{9\sqrt[3]{x^4}}$$

(a) Find the critical points of f. Classify each as a local maximum, local minimum, or neither.

(b) Find any inflection points of f. Be sure to also consider where $f''(x)$ is undefined when identifying possible inflection points.

(c) Use the information in parts (a)–(b) to sketch f so that f passes through the point $(0, 5)$. Label all critical points and inflection points.



3. (10 pts) Find the equation of the tangent line to the curve $2x^2y - y^2 = x$ at the point $(1, 1)$.

4. (20 pts) Consider $f(x) = \dfrac{e^x}{x^2 - 1}$.

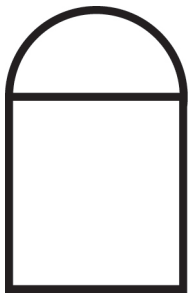
(a) Using limits, complete the definition: "The line $y = k$ is a *horizontal asymptote* for the graph of f if . . . "

(b) Find all horizontal asymptotes, if any exist, for the graph of $f(x)$. Justify your answer using limits.

(c) Using limits, complete the definition: "The line $x = a$ is a *vertical asymptote* for the graph of f if . . . "

(d) Find all vertical asymptotes, if any exist, for the graph of $f(x)$. Justify your answer using limits.

5. (15 pts) A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20 feet, find the dimensions of the rectangle that will produce the largest area for the window. Neglect the thickness of the frame. (Helpful formulas related to circles: Area $= \pi r^2$ & Circumference $= 2\pi r$.)



(a) What quantity are you trying to optimize? Are you trying to minimize it or maximize it?

(b) What is the objective function for the quantity you are trying to optimize?

(c) Find the constraint equation and use it to rewrite the objective function from (b) as a function of one variable.

(d) Find the critical point(s) of the objective function. Verify that you have the desired max or min.

(e) What are the dimensions of the rectangle?

6. (a) (7 pts) Rewrite $f(x) = \csc(\arccos(2x))$ as an algebraic expression - no trigonometric or inverse trigonometric functions.

(b) (5 pts) Use your answer from (a) to find $f'(x)$. Note: f' should also be an algebraic expression. Simplify your answer.

7. (7 pts) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ when $y = \sqrt{x}^{\,x}$.