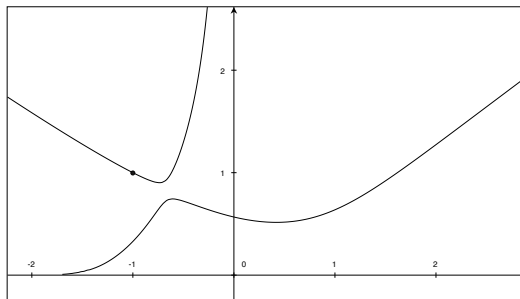


Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

1. (20 pts) Let  $x^3 - 2xy^2 - y = \ln y$ .

- (a) Use implicit differentiation to find a formula for  $\frac{dy}{dx}$ .



- (b) What is the equation of the line tangent to the curve at the point  $(-1, 1)$ ?

2. (30 pts) The table below provides information about the functions  $f$ ,  $g$ , and their derivatives.

$x$	$f$	$f'$	$g$	$g'$
-1	2	-3	4	-2
0	1	0	3	-1
1	-1	1	-3	-5
3	-1	0	5	4

Suppose that:

$$h(x) = f(x)g(x), \quad k(x) = (x^2 + g(x))^3, \quad \text{and } j(x) = \frac{e^{g(x)}}{\cos x}.$$

Use the table to compute the following values:

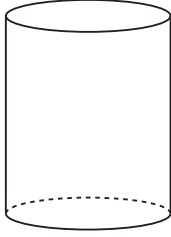
(a)  $h(3)$

(b)  $h'(1)$

(c)  $k'(-1)$

(d)  $j'(0)$

3. (20 pts) Consider a right circular cylinder whose total surface area, including the top and bottom circular ends, is  $240\pi \text{ cm}^2$ . What dimensions (radius and height) produce a cylinder with maximum volume? What is the maximum volume?



- (a) What quantity are you trying to optimize? Are you trying to minimize it or maximize it?
- (b) What is the objective function for the quantity you are trying to optimize?
- (c) Find the constraint equation and use it to rewrite the objective function from (b) as a function of one variable.
- (d) Find the critical point(s) of the objective function. Verify that you have the desired max or min.
- (e) What are the optimal dimensions of the cylinder?

4. (30 pts) Evaluate the following limits.

(a)  $\lim_{x \rightarrow \infty} \frac{4x^7 - 3x^5 + 2x - 8}{9x^3 + x^6 - 12x^7 + 1}$

(b)  $\lim_{x \rightarrow 0} \frac{\arcsin x}{x}$

(c)  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

**BONUS** Use a reference triangle to write  $f(x) = \cot(\arcsin(1 - x))$  as an algebraic expression, i.e., one not involving any trigonometric functions.