TEST 2

Math 105
11/9/12

Name: ______________________
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*3 points*) Solve the following differential equation $y' = 5y$ with initial condition $y(0) = R$.

$$y = Re^{5x}$$

2. (*14 points*)

x	$f(x)$	$g(x)$	$j(x)$	$f'(x)$	$g'(x)$	$j'(x)$
-2	0	1	-1	3	2	1
-1	1	3	2	-1	3	0
0	2	1	1	2	-2	2
1	3	1	-1	0	3	1
2	-2	2	1	3	0	3
3	-1	1	-1	1	-2	2

a. (*7 pts*) $H(x) = f(j(x)) + g(x^2 - 1)$. Find $H'(2)$.

$$H'(x) = f'(j(x)) \cdot j'(x) + g'(x^2 - 1) \cdot 2x$$

$$H'(2) = 0 * 3 + (-2) * 4 = -8$$

b. (*7 pts*) $F(x) = \dfrac{g(x)^2}{(x+1)j(x)}$. Find $F'(0)$.

$$F'(x) = \frac{2g(x)g'(x)(x+1)j(x) - (j(x) + (x+1)j'(x))g(x)^2}{(x+1)^2 j(x)^2}$$

$$F'(0) = \frac{2 * 1 * (-2)(1)(1) - [(1 + 1 * 2) * 1}{1 * 1} = -7$$

3. (*20 points*) Find y'.

a. (*10 pts*) $y = \dfrac{\sin^4(x)(3x^4 - 2x + 5)^3 e^{3x}}{(x+1)^2(2x-3)^5}$ using logarithmic differentiation.

$$y' = \left(\frac{4\cos(x)}{\sin(x)} + \frac{36x^3 - 6}{3x^4 - 2x + 5} + 3 - \frac{2}{x+1} - \frac{10}{2x-3}\right) \frac{\sin^4(x)(3x^4 - 2x + 5)^3 e^{3x}}{(x+1)^2(2x-3)^5}$$

b. (*10 pts*) $y = \sqrt[4]{\tan^2(2x+1)} + 2^{cos(3x)} - \arcsin(3x^2)$

$$\frac{1}{4}(\tan^2(2x+1))^{-3/4} \cdot 2\tan(2x+1) \cdot \sec^2(2x+1) \cdot 2 + \ln(2)2^{\cos(3x)} \cdot (-3\sin(3x)) - \frac{6x}{\sqrt{1 - 9x^4}}$$

Or see that the original function can be simplified: $y = \sqrt[2]{\tan(2x+1)} + 2^{cos(3x)} - \arcsin(3x^2)$.

$$\frac{1}{2}(\tan(2x+1))^{-1/2} \cdot \sec^2(2x+1) \cdot 2 + \ln(2)2^{\cos(3x)} \cdot (-3\sin(3x)) - \frac{6x}{\sqrt{1 - 9x^4}}$$

These are equivalent.

4. (*11 points*)

a. (*9 pts*) For the equation $\sin(xy^2) = 3x^2 + 3y^3 - 3$ use implicit differentiation to find $\frac{dy}{dx}$.

$$\cos(xy^2) \cdot (y^2 + 2xyy') = 6x + 9y^2y'$$

$$y' = \frac{6x - \cos(xy^2)y^2}{\cos(xy^2)2xy - 9y^2}$$

b. (*2 pts*) Determine $\frac{dy}{dx}$ at the point (0,1).

$$y' \mid_{(0,1)} = \frac{1}{9}$$

5. (*18 points*) Find the antiderivative of the given function.

a. (*6 pts*) $h(x) = x^3 - e^{6x} + 4\cos(x) - \frac{4}{x}$.

$$\frac{x^4}{4} - \frac{e^{6x}}{6} + 4\sin(x) - 4\ln(x) + C$$

b. (*6 pts*) $f(x) = \frac{2}{4 + x^2}$.

$$\arctan\left(\frac{x}{2}\right) + C$$

c. (*6 pts*) $g(x) = \frac{2x}{4 + x^2}$.

$$\ln(4 + x^2) + C$$

6. (*16 points*) Evaluate the following limits. Only use L'Hôpital's rule when appropriate. Show your work!!

a. (*8 pts*) $\lim_{x\to\infty} \frac{(\ln x)^2}{x}$

$$\text{Type: } \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \lim_{x\to\infty} \frac{2\ln(x)\cdot(x^{-1})}{1} = \lim_{x\to\infty} \frac{2\ln(x)}{x} = \lim_{x\to\infty} \frac{2x^{-1}}{1} = \lim_{x\to\infty} \frac{2}{x} = 0$$

b. (*8 pts*) $\lim_{x\to\infty} \left(1+\frac{k}{x}\right)^x$, for k a constant.

$$y = \lim_{x\to\infty} \left(1+\frac{k}{x}\right)^x$$

$$\ln(y) = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right), \text{ Type } \infty\cdot 0$$

$$\ln(y) = \lim_{x\to\infty} \frac{\ln\left(1+\frac{k}{x}\right)}{\frac{1}{x}}, \text{ Type } \frac{0}{0}$$

$$\ln(y) = \lim_{x\to\infty} \frac{\ln\left(1+\frac{k}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{-kx^{-2}}{1+\frac{k}{x}}}{-x^{-2}} = \lim_{x\to\infty} \frac{k}{1+\frac{k}{x}} = k$$

$$y = e^k$$

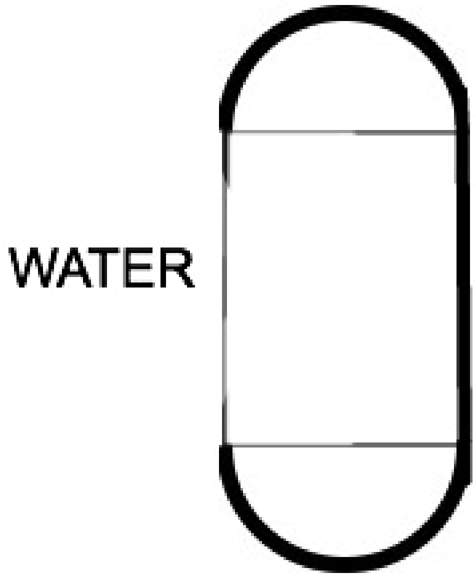
7. (*3 points*) Does $f(x) = \frac{5x^3+6x-1}{2x^3+10}$ have a horizontal asymptote? If so, where is it?

$$\lim_{x\to\infty} \frac{5x^3+6x-1}{2x^3+10}, \text{ Type } \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{5x^3+6x-1}{2x^3+10} = \lim_{x\to\infty} \frac{15x^2+6}{6x^2} = \lim_{x\to\infty} \frac{30x}{12x} = \frac{5}{2}$$

Yes, the function has a horizontal asymptote at $y = \frac{5}{2}$.

8. (*15 points*) A farmer wishes to build a corral for his cows in the shape below (rectangle with semi-circles on the ends). She has 1000ft of fencing and she won't fence-in the straight edge along the water (just bolded sides). What is the maximal area for the corral?



Label Water side x and the top of the rectangle y. The objective is the area of the corral. The constraint is the 1000ft of fencing for the perimeter.

$$A = xy + \frac{\pi y^2}{4}$$

$$1000 = x + \pi y$$

Therefore, $x = 1000 - \pi y$ and $A = (1000 - \pi y)y + \frac{\pi y^2}{4}$.

$$A = 1000y - \frac{3\pi}{4}y^2$$

$$A' = 1000 - \frac{6\pi}{4}y$$

Set the derivative to 0.

$$\frac{6\pi}{4}y = 1000 \rightarrow y = \frac{4000}{6\pi}$$

Check to see if this is a max or min.

$$A'' = -\frac{6\pi}{4}$$

Since the second derivative is always negative, then $y = \frac{4000}{6\pi}$ is a local maximum.
Check for global maximums on the interval [0,318.3].
A(0) = 0 sq. ft., A($1000/\pi$) = 79,577.47 sq. ft (this is just a circle), A($\frac{4000}{6\pi}$ ≅212.2) = 106,103 sq. ft.
The maximal area is 106,103 sq. ft.