

Name: _____

Exam 2 – Math 105D

Show all your work to receive full credit for a problem. There are a total of 72 points on this test. Good luck!

1. (4 pts. each) If $p(1) = 3$ and $p'(1) = 2$, find $f'(1)$ for the following:

(a) $f(x) = \frac{\ln x}{p(x)} + p(x^5)$

(b) $f(x) = e^{p(x)} \cos(x - 1)$

(c) $f(x) = \arctan(p(x))$

2. (4 pts each, 12 points total) Are the following statements True or False? If a statement is true, explain how you know. If a statement is false, give a counterexample.

(a) A local minimum of f occurs at a critical point of f .

(b) If $f'(x) = g'(x)$ for all x , then $f(x) = g(x)$ for all x .

(c) Let f and g be two functions whose second derivatives are defined. Then

$$(fg)'' = fg'' + f''g.$$

3. (6 pts.) For each of the following pairs of functions, determine if F is an antiderivative of f . Explain your answers.

(a) $F(x) = xe^{\tan x} + \pi$, $f(x) = e^{\tan x}(1 + x \sec^2 x)$.

(b) $F(x) = \sin^{11}(x^3) + 2$, $f(x) = 33x^2 \cos^{10}(x^3)$.

(c) $F(x) = \arctan(\sin x)$, $f(x) = \frac{\sin x \cos x}{1 + x^2}$.

4. (6 pts each.) Evaluate the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\cos(2x) - \cos(6x)}{-4x^2}$

(b) $\lim_{x \rightarrow \infty} \frac{7(\ln x)^2}{3x}$

5. (6 pts.) Assume y is a differentiable function of x and that $x^2y^2 + x \sin y = 7$. Find $\frac{dy}{dx}$.

6. (12 pts.) Some airlines have restrictions on the size of items of luggage that passengers are allowed to take with them. Suppose that one has a rule that the sum of the length, width and height of any piece of luggage must be less than or equal to 210 cm. A passenger wants to take a box of the maximum allowable volume (so assume it will also be of the biggest size allowable). If the length of the box is twice the width, what should the dimensions of the box be?