1. Find $\frac{dy}{dx}$ in each case below.

1A) $y = \pi^x + 3^x + x^3 + x^\pi + e^\pi + \frac{x^3 + 3x^2}{5x + 11}$

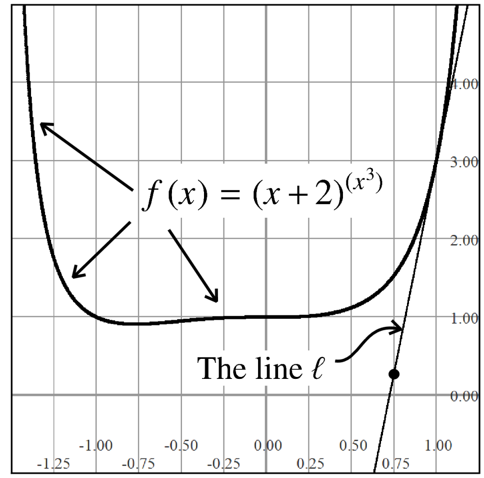
1B) $y = \tan(x^3 + \log_2(x))$

1C) $y = \cos^{-1}(x^3) + (\cos(x^3))^{-1} + \cos^2(x^3)$

Hint You'll find this much easier if you rewrite the first and third terms without the bad notation they are written in. *Hint*^{2} In particular, does the first term represent an inverse function or 1-over-something?

1D) $y = \ln\left(\arctan(\sqrt{x^2 + 4x + 2})\right)$

2. Consider the function $y = f(x) = (x+2)^{(x^3)}$. The accompanying plot shows the graph of this function along with the line ℓ tangent to the graph at the point $(1, 3)$.



2A: Use logarithmic differentiation to find y'.

2B: Use the answer to 2A to find the slope of the line ℓ; give the answer to **at least seven** digits after the decimal point.

2C: Use the graph to estimate the slope of the line ℓ. *Hint:* The "fat point" has coordinates $\approx (0.75, 0.28)$.

2D: Do the answers to 2B and 2C agree (at least to two decimal places?) Explain why not if they don't.

2E: There appears to be a local minimum for $f(x)$ between $x_L = -1$ and $x_R = -0.5$. Use the `minimum` function on your calculator with these two x's as Left Bound and Right Bound, respectively, to find the x_m coordinate at which this minimum occurs; write your answer to **at least seven** places after the decimal point.

2F: Since x_m is a stationary point, you expect $f'(x_m)$ to be what? [ ]

And using the answers to 2A and 2E, what is $f'(x_m)$? [ ] (write your answer to as many places as your calculator displays:

Did the calculator find the exact value of x_m or just a good approximation? [ ] Explain your answer.

3. Consider $\lim_{x\to 0^+} \dfrac{\sin 5x}{4x}$.

3A: What "form" does this limit have? [ ] Is it "indeterminate"? [ ]

3B: Find the limit; use "LH" if and only if it's appropriate. Write down all the steps in the accepted notation.

3C: Make a small table with three well-chosen x values that support your conclusion in 3B.

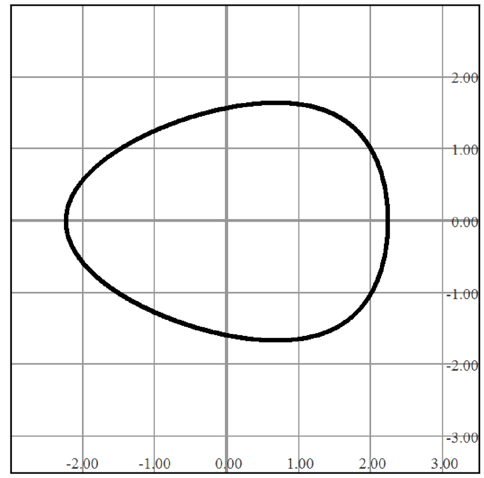
4. BONUS QUESTION! (To get bonus points, all parts have to be essentially correct!) Consider $\lim_{x\to\infty} \dfrac{1.01^x + x^2}{1.01^x + 100x}$.

4A: What "form" does this limit have? [ ] Is it "indeterminate"? [ ]

4B: Find the limit; use "LH" at each stage if and only if it's appropriate. Write down all the steps in the accepted notation.

4C: Use your calculator to plot $\dfrac{1.01^x + x^2}{1.01^x + 100x}$ in the window $[0, 3000] \times [0, 10]$ and make an excellent facsimile here. Does the plot support your conclusion in 4B? Explain why or why not.

5. The equation $xy^2 + 10 = 2x^2 + 4y^2$ implicitly defines y as a function of x. The graph formed by the solutions of this equation look like the cross section of an egg, as shown here.



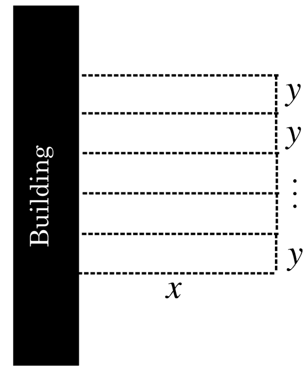
5A: Find y' using implicit differentiation.

5B: The plot suggests that $(2, 1)$ is a solution of the equation $xy^2 + 10 = 2x^2 + 4y^2$. Verify this.

5C: What is the slope of the graph of the egg at $(2, 1)$?

5D: *BONUS!*. Find the explicit formula for the function whose graph is the bottom half of the egg in the figure.

6. Five animal pens are to be fenced-in next to a very long building as in the accompanying figure; each pen has fencing on three sides and the building serves as the fourth side of each pen (no fence needed on the fourth sides). The pens are all the same size and *together* must enclose a total area of 2400 square feet. What should the dimensions x and y be in order to minimize the total length of the fence required for this project? [Answer the following parts first].



6A: In particular: What is the **objective function** here?

6B: What is (or are) the **constraint equation(s)**?

6C: Rewrite the objective function in terms of just one variable, and give the result here:

6D: Graph the (rewritten) objective function on your calculator after choosing an appropriate "window", and make an excellent sketch of the result in the box provided. LABEL the axes in your picture and tell me what "window" (Xmin, Xmax, etc) your sketch represents. Explain why your plot suggests we are indeed finding a minimum.

Your Window:

6E: Solve the problem using calculus to locate the appropriate minimum point on your graph.

6F: Finally then, what are the values of x, y and the total amount of fence required? Label your answers.