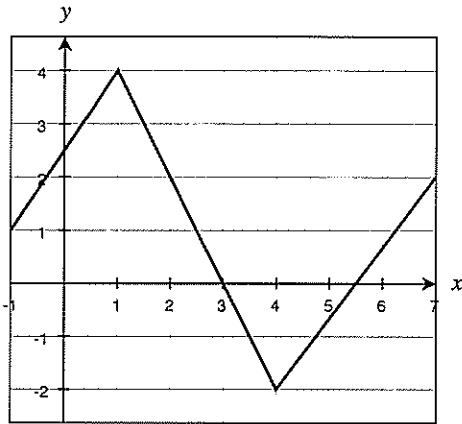


Name: Solutions

Math 105: Fall 2013
Exam 2: November 8

Correct answers accompanied by incorrect or incomplete work will not receive full credit.
There is a unit circle on the last page of the exam.

1. (14 points) Consider a function f that has the following graph.



Let $h(x) = \frac{f(x)}{2^x}$.

(a) Write an expression for $h'(x)$.

$$h'(x) = \frac{f'(x)2^x - (\ln 2)2^x f(x)}{(2^x)^2}$$

(b) Write an equation of the line tangent to h at $x = 3$.

$$f(3) = 0, f'(3) = -2, h(3) = \frac{f(3)}{2^3} = 0$$

$$h'(3) = \frac{(-2)(2^3) - (\ln 2)2^3(0)}{(2^3)^2} = \frac{-2(8)}{8^2} = \frac{-2}{8} = -\frac{1}{4}$$

$y = -\frac{1}{4}(x-3)$

2. (12 points) Consider a function g that has the following values and whose derivative has the following values:

x	$g(x)$	$g'(x)$
$-\pi/3$	1	-1
$-\pi/6$	2	-2
0	3	-3
$\pi/6$	4	-4
$\pi/3$	5	-5
$5\pi/3$	6	-6
$11\pi/6$	7	-7

Let $m(x) = g(\arctan x)$.

(a) Write an expression for $m'(x)$.

$$m'(x) = g'(\arctan x) \cdot \frac{1}{1+x^2}$$

(b) Find $m'(-\sqrt{3})$.

$$\arctan(-\sqrt{3}) = -\pi/3$$

b/c $\frac{\sin(-\pi/3)}{\cos(-\pi/3)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

$$m'(-\sqrt{3}) = g'(\arctan(-\sqrt{3})) \cdot \frac{1}{1+(-\sqrt{3})^2}$$

$$= g'(-\pi/3) \cdot \frac{1}{1+3} = -1 \cdot \frac{1}{4} = \boxed{-\frac{1}{4}}$$

3. (20 points) Find y' in 2 of 3 of the following. If you do more than two, then clearly mark which two you want graded. If you don't, the worst two will be chosen for you.

(a) $y = e^{\arcsin x} + (4x^2 + 6x^3) \cos x$

(b) $y = \sin^{11}(x^3) + 14 \ln(27x^2 + 5x) = (\sin(x^3))^{11}$

(c) $y = \sqrt[3]{\frac{14}{x} + e^x \sin x} = (14x^{-1} + e^x \sin x)^{1/3}$

(a) $y' = e^{\arcsin x} \cdot \frac{1}{\sqrt{1-x^2}} + (8x + 18x^2) \cos x + (4x^2 + 6x^3)(-\sin x)$

(b) $y' = 11(\sin(x^3))^{10} (\cos(x^3)) \cdot 3x^2 + 14 \cdot \frac{1}{27x^2 + 5x} (54x + 5)$

(c) $y' = \frac{1}{3} (14x^{-1} + e^x \sin x)^{-2/3} [-14x^{-2} + e^x \sin x + e^x \cos x]$

4. (10 points) Find $\frac{dy}{dx}$ in 1 of 2 of the following. If you do more than one, then clearly mark which one you want graded. If you don't, the worst will be chosen for you.

(a) $y = \frac{(\cos^2 x)(x^3 - 7x)(e^{3x^4})}{(2^x)}$

(b) $y^3 + x \sin y = 15x$

(a) use logarithmic differentiation.

$$\ln y = \ln((\cos x)^2) + \ln(x^3 - 7x) + \ln(e^{3x^4}) - \ln(2^x)$$

$$\ln y = 2 \ln(\cos x) + \ln(x^3 - 7x) + 3x^4 \underbrace{\ln e}_1 - x \ln 2$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{-2 \sin x}{\cos x} + \frac{3x^2 - 7}{x^3 - 7x} + 12x^3 - \ln 2$$

$$\frac{dy}{dx} = \frac{(\cos^2 x)(x^3 - 7x)e^{3x^4}}{2^x} \left[\frac{-2 \sin x}{\cos x} + \frac{3x^2 - 7}{x^3 - 7x} + 12x^3 - \ln 2 \right]$$

(b) $3y^2 \frac{dy}{dx} + \sin y + x \cos y \frac{dy}{dx} = 15$

$$3y^2 \frac{dy}{dx} + x \cos y \frac{dy}{dx} = 15 - \sin y$$

$$(3y^2 + x \cos y) \frac{dy}{dx} = 15 - \sin y$$

$$\frac{dy}{dx} = \frac{15 - \sin y}{3y^2 + x \cos y}$$

5. (8 points) Is $y = 2x^5$ a solution to the Initial Value Problem

$$y' = \frac{5y}{x}, \quad y(1) = 2?$$

Justify your answer.

If $y = 2x^5$ then $y' = 10x^4$.

plug these into $y' = \frac{5y}{x}$ and see if equality still holds.

$$10x^4 \stackrel{?}{=} \frac{5(2x^5)}{x}$$

$$10x^4 \stackrel{?}{=} 10x^4 \quad \checkmark$$

We also check $y(1) = 2(1)^5 = 2 \quad \checkmark$

So yes $y = 2x^5$ is a solution to the IVP

6. (15 points) Let f be a function with first and second derivatives given below:

$$f'(x) = \frac{1 - \ln x}{x - 4}, \quad f''(x) = \frac{-2 + \frac{4}{x} + \ln x}{(x - 4)^2}$$

Find the *critical points* of f on the interval $(0, \infty)$. Give the x -values exactly, *not* as decimal approximations. Classify each critical point as a local maximum, local minimum, or neither.

critical points when $f' = 0$ or when f' is undef.

$$f'(x) = 0 = \frac{1 - \ln x}{x - 4} \Rightarrow 1 - \ln x = 0$$

$$\ln x = 1$$

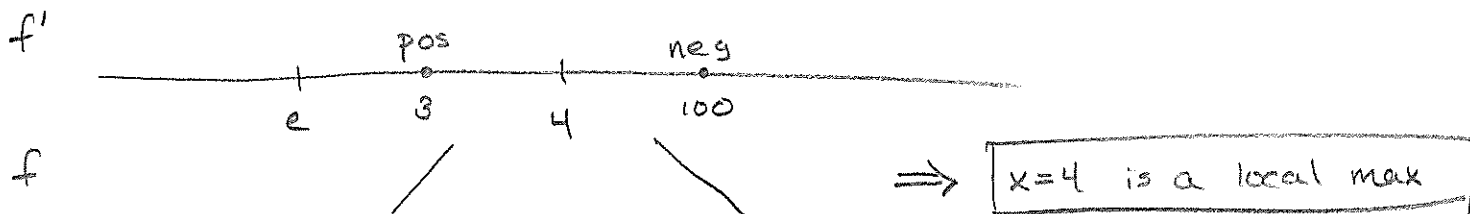
$$x = e$$

On the interval $(0, \infty)$ f' is only undefined when $x = 4$.

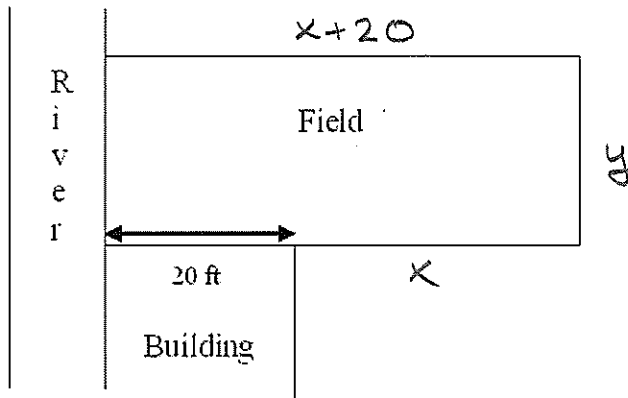
The two critical points are $\boxed{x = e, x = 4}$

$$f''(e) = \frac{-2 + \frac{4}{e} + \ln e}{(e - 4)^2} = \frac{\text{pos}}{\text{pos}} = \text{pos} \Rightarrow f \text{ is cc } \uparrow$$

$$\Rightarrow \boxed{x = e \text{ is a local min}}$$



7. (20 points) A rectangular field as shown is to be bounded by a fence. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fencing. You can assume that fencing is not needed along the river and building. (Be sure to show how you know you have found the maximum.)



maximize area.

$$A = (x+20)y$$

$$1000 = x + y + x + 20$$

$$1000 = 2x + y + 20$$

$$980 - 2x = y$$

$$A = (x+20)(980-2x)$$

$$A = 980x - 2x^2 + 19600 - 40x$$

$$A = -2x^2 + 940x + 19600$$

$$A' = -4x + 940$$

$$-4x + 940 = 0$$

$$4x = 940$$

$$x = 235$$

$$A'' = -4$$

$$A''(235) = -4 \Rightarrow A \llcorner \downarrow$$

$\Rightarrow x=235$ is
a local max

$$y = 980 - 2(235) = 510$$

so the field is 510 ft by 235 ft

8. (1 point) What is your favorite food?

ice cream!

