

Name: Key

1. An engineer has a homogeneous system of 45 equations with 50 unknowns.

(a) What is the minimal number of free variables that the system could have?

The maximal rank is 45, so the minimal number of free variables is $50 - 45 = 5$

(b) Could an associated non-homogeneous system be consistent? Why or why not?

Yes, the system could have rank 45 so the system could have a pivot in every row & be consistent.

(c) Could an associated non-homogeneous system be consistent with exactly one solution? Why or why not?

No, this system will always have free variables. There cannot be exactly one solution.

2. Find the determinant of the matrix $\begin{bmatrix} 2 & 1 & 4 & 0 \\ -2 & 0 & -3 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & 2 & 0 & -1 \end{bmatrix}$.

$$\begin{aligned} & 2 \begin{vmatrix} 2 & 4 & 0 \\ -2 & -3 & 1 \\ -1 & 2 & 0 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 & 4 \\ -2 & 0 & -3 \\ -1 & 0 & 2 \end{vmatrix} \\ &= 2(-1) \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} - 1(-1) \begin{vmatrix} -2 & -3 \\ -1 & 2 \end{vmatrix} \\ &= 2(-1)(8) - 1(-1)(-7) \\ &= -16 - 7 = \boxed{-23} \end{aligned}$$

3. What must be true of h and k for the determinant of $\begin{bmatrix} 1 & 0 & k \\ 0 & h & 0 \\ 1 & 0 & -1 \end{bmatrix}$ to be zero?

$$\text{determinant is } h \cdot \begin{vmatrix} 1 & k \\ 1 & -1 \end{vmatrix} = h(-1-k) = 0$$

So ~~either~~ either $k = -1$
or $h = 0$