Name:____

Exam 2 – Math 105C

Show all your work to receive full credit for a problem. There are a total of 72 points on this test. Good luck!

1. (2 points each) The following problems have blanks that you need to fill in with the correct answer. You do not need to show work for these problems.

   Here is an example question like questions (a)-(g), with a correct answer to illustrate: If $F'(x) = f(x)$, we say that F is an antiderivative of f.

   (a) The number of cells in a tumor doubles every 6 weeks starting with 8 cells. The best function to describe this phenomenon is ________________________.

   (b) A spring hangs from the ceiling at equilibrium with a mass attached to its end. Suppose you pull downward on the mass and release it 10 inches below its equilibrium position. Which two functions could best describe the distance in inches of the mass from its equilibrium after you let it go?

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   (c) If the derivative of a function f doesn't exist at a point c, we say that c is a ________________________ point of f.

   (d) If we evaluate a limit and get something of the form $\frac{0}{0}$, we say we have an ________________________ form.

   (e) If f and g are functions such that $(f \circ g)(x) = (g \circ f)(x) = x$, then we say that they are ________________________ of each other.

   (f) If c is a point such that $f'(c) = 0$ and $f''(c)$ is negative, we can say that f is a local ________________________.

   (g) The method used in the part (f) is known as the ________________________ ________________________ test.

2. (3 points each) Suppose f is a function with the following values at 0 and 1.

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	$\frac{1}{5}$

Compute the derivatives of the following functions at the points indicated.

(a) $\sqrt[3]{x}f(x)$ at $x = 1$.

(b) $f(1 - 5\tan x)$ at $x = 0$.

(c) $\dfrac{f(x)}{2 + \cos x}$ at $x = 0$.

(d) $10\sin\left(\frac{\pi x}{2}\right)(f(x))^2$ at $x = 1$.

3. (3 points each) Differentiate the following. Use logarithmic differentiation where necessary.

(a) $f(x) = \log_7(e^x)$

(b) $f(x) = \ln(1 + \ln(1 + x))$

(c) $f(x) = e^{e^{e^x}}$

(d) $f(x) = (\sin x)^{\cos x}$

4. (6 points) Assume y is a differentiable function of x and satisfies the equation

$$x \arcsin y = 1 + x^2.$$

Compute $\dfrac{dy}{dx}$.

5. (3 points each) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{100x^3 - 3}{x^4 - 2}$.

(b) $\displaystyle\lim_{x\to 0} \frac{1 - x - e^{-x}}{1 - \cos x}$.

6. (10 points) A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot wide decks along either side and 10-foot wide decks at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.