

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. In other words, correct answers accompanied by incorrect or incomplete work will not receive full credit.

1. (30 pts) Evaluate the following integrals.

(a)  $\int e^x e^{2x} dx$

(b)  $\int \arcsin x dx$

(c)  $\int \frac{dx}{(1+x^2)^2}$

(d)  $\int \frac{2x^2}{1+x} dx$

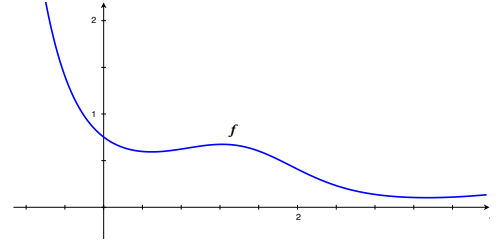
2. (15 pts) Suppose you want to analyze the fishing industry in a small coastal town in Maine. Each day, the boats bring back at least 2 tons of fish but never more than 8 tons. Using the probability density function given below, determine the probability that the catch is between 5 and 7 tons.

$$f(x) = \begin{cases} 0.04x & \text{for } 2 \leq x \leq 6 \\ 0.6 - 0.06x & \text{for } 6 < x \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

3. (10 pts) The following integral is improper. **SET UP, BUT DO NOT EVALUATE** an appropriate expression involving integral(s) *with limit(s)* to demonstrate that you know how to proceed in evaluating this improper integral.

$$\int_0^{\infty} \frac{dx}{(x-1)^3}$$

4. (15 pts) Suppose that  $P_2(x) = a + bx + cx^2$  is the second degree Taylor polynomial for the function  $f$  about  $x = 0$ . Consider the graph of  $f$  given below, what can you say about the signs of  $a$ ,  $b$ , and  $c$ ? **You must explain your answers.**



5. (20 pts) Consider  $\int_1^{\infty} \frac{e^{2+\sin x}}{x^2} dx$ . Use a comparison to determine if this improper integral converges or diverges? If it converges, find an upper bound for its value. **You must explain why you selected your particular comparison integral.** (Hint: to start, think about what you can say about the size of  $2 + \sin x$ .)

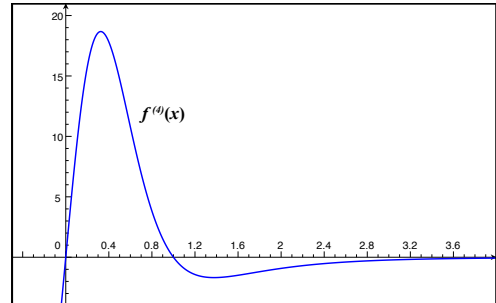
6. (10 pts) If  $f(x) = 4 \arctan x$ , then  $f(1) = \pi$ . The third order Maclaurin Polynomial for  $f(x) = 4 \arctan x$  is given by  $P_3(x) = 4x - \frac{4x^3}{3}$ .

(a) Give an approximation for  $\pi$  using  $P_3(x)$ .

(b) Use the fact that  $f^{(4)}(x) = -\frac{96x(x^2 - 1)}{(x^2 + 1)^4}$ , shown on the right, to determine the *maximum possible error* when using  $P_3(x)$  to estimate  $f(x)$  on the interval  $[0, \frac{3}{2}]$ .

Recall Taylor's Theorem says:

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!} |x - x_0|^{n+1} \text{ for } x_0 \text{ in } [a, b]$$



**You must explain your method in finding a value for  $K_{n+1}$ .**

**BONUS:** What is the actual maximum error on  $[0, \frac{3}{2}]$  when using  $P_3(x)$  to approximate  $f(x)$ ?

*Trigonometric Identities*

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta = 1 + \tan^2 \theta$
- $\sin(2\theta) = 2 \sin \theta \cos \theta$
- $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$
- $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$

*Some antiderivative rules:*

- $\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C$
- $\int \csc \theta \, d\theta = \ln |\csc \theta - \cot \theta| + C$
- $\int \tan \theta \, d\theta = -\ln |\cos \theta| + C$
- $\int \cot \theta \, d\theta = -\ln |\csc \theta| + C$
- $\int \csc^2 \theta \, d\theta = -\cot \theta + C$