

MATH 205A,B - LINEAR ALGEBRA
FALL 2015

QUIZ 7

NAME:

Section:(Circle one) A(8 : 00) B(9 : 30)

Show ALL your work CAREFULLY.

(a) Find the coordinate vector $[\vec{x}]_{\mathbf{B}}$ relative to the basis $\mathbf{B} = \{\vec{b}_1, \vec{b}_2\}$ where

$$\vec{b}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \text{ and } \vec{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

The change of basis matrix is $P_{\mathbf{B}} = \begin{bmatrix} -1 & -1 \\ 3 & 2 \end{bmatrix}$ and $\vec{x} = P_{\mathbf{B}}[\vec{x}]_{\mathbf{B}}$. Now, $P_{\mathbf{B}}^{-1} = \frac{1}{(-2+3)} \begin{bmatrix} 2 & 1 \\ -3 & -1 \end{bmatrix}$.

It follows that

$$[\vec{x}]_{\mathbf{B}} = \begin{bmatrix} 2 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}.$$

(b) Find a basis for the column space $\text{Col}A$ of the matrix A where

$$A = \begin{bmatrix} 1 & 0 & -1 & -1 & 1 \\ 1 & 1 & 0 & -1 & 0 \\ -1 & 2 & 3 & 2 & -1 \end{bmatrix}.$$

What is the dimension of $\text{Nul}A$?

Note that

$$A \sim \begin{bmatrix} 1 & 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & 2 & 3 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 2 & 2 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

It follows that the 1st, 2nd, and 4th columns have pivots thus

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

form a basis for $\text{Col}A$. Now, $\dim \text{Col}A = 3$ and $\dim \text{Col}A + \dim \text{Nul}A = 5$, it follows that $\dim \text{Nul}A = 2$.