Math 205 A B Quiz 06 gold page 1 11/02/2012 **Name** Suggested Solutions

1. Find the determinant of the following matrix by hand, showing all your steps (intermediate results) along the way. Make good use of 0's. Circle your final answer.

$$\begin{bmatrix} 2 & 4 & 0 & 5 \\ 5 & 0 & 0 & 11 \\ 12 & 8 & 3 & 7 \\ 9 & 0 & 0 & 6 \end{bmatrix} = (\text{by } 3^{rd} \text{ column}) \; 0|A_{13}| - 0|A_{23}| + 3\begin{vmatrix} 2 & 4 & 5 \\ 5 & 0 & 11 \\ 9 & 0 & 6 \end{vmatrix} - 0|A_{43}|$$

$$= 3\begin{vmatrix} 2 & 4 & 5 \\ 5 & 0 & 11 \\ 9 & 0 & 6 \end{vmatrix} = (3)(-4)\begin{vmatrix} 5 & 11 \\ 9 & 6 \end{vmatrix} = -12 \cdot -69 = 828$$

(there are many "paths" to this solution, but this seems easiest)

2. Let $B = \begin{bmatrix} 1 & 2 & 4 \\ a & b & c \\ r & s & t \end{bmatrix}$; suppose det$(B)$=5

Find the determinant of each of the following matrices, and under each matrix *write the reason/rule/fact about determinants* of matrices you used to find the det. (eg, "swapping rows changes the sign of the det" or "the determinant of the derivative of a matrix is the matrix of its integral" (this second fact is nonsense)

2a) $M = \begin{bmatrix} 1 & a & r \\ 2 & b & s \\ 4 & c & t \end{bmatrix}$

$|M| = |B^T| = |B| = 5$

b/c the dets of a matrix and its transpose are the same.

2b) $N = \begin{bmatrix} 1 & 2 & 4 \\ a & b & c \\ r+2 & s+4 & t+8 \end{bmatrix}$ $|N| = 5$ b/c

N is obtained from B by "$r_3 \leftarrow r_3 + 2r_1$". This row op. doesn't change the underlying dets.

2c) $Q = \begin{bmatrix} 10 & 20 & 40 \\ a & b & c \\ r & s & t \end{bmatrix}$

multiplying a row of B by a non zero constant α changes the det of the resulting matrix by α.

so $\det(Q) = 10\det(B) = 50$

2d) $R = \begin{bmatrix} 10 & 20 & 40 \\ a & b & c \\ 1 & 2 & 4 \end{bmatrix}$ this matrix cannot be obtained from B by elementary row ops.

BUT if we subtract 10 copies of row 3 from row 1 in R we get

$$|R| = \begin{vmatrix} 0 & 0 & 0 \\ a & b & c \\ 1 & 2 & 4 \end{vmatrix} = 0.$$

2e) $S = 4B$ $\det(S) = \det(4B)$ but NOT $4\det(B)$!

this is because EACH row of B gets multiplied by 4, hence $\det(4B) = 4 \cdot 4 \cdot 4 \cdot \det(B)$
$= 64 * 5 = 320$

3) Suppose the following elementary row operations turn the matrix A into U:
First, rows r_1 and r_2 are swapped. Second, $r_3 \leftarrow r_3 + 10r_2$, Third, row 1 is multiplied by 4. The matrix U is a 3×3 upper triangular matrix, with main diagonal entries 4, 3, and 2.

3A) What is det(A)? Explain. represent these row ops by elementary matrices E_1, E_2 & E_3, respectively.

We're told that $E_3 E_2 E_1 A = U$.

so: $\det(E_3 E_2 E_1 A) = \det(U)$

thus $\det(E_3)\det(E_2)\det(E_1)\det(A) = \det(U)$

$$(4)(1)(-1) \cdot |A| = (4 \cdot 3 \cdot 2) \Rightarrow |A| = \frac{24}{-4} = -6$$

3B) Is A invertible? Explain how you know.

A is invertible b/c $\det(A) \neq 0$.