

1. Find the determinant of the following matrix by hand, showing all your steps (intermediate results) along the way. Make good use of 0's. Circle your final answer.

$$\begin{bmatrix} 2 & 4 & 0 & 5 \\ 5 & 0 & 0 & 11 \\ 12 & 8 & 3 & 7 \\ 9 & 0 & 0 & 6 \end{bmatrix}$$

2. Let  $B = \begin{bmatrix} 1 & 2 & 4 \\ a & b & c \\ r & s & t \end{bmatrix}$ ; suppose  $\det(B)=5$

Find the determinant of each of the following matrices, and under each matrix *write the reason/rule/fact about determinants* of matrices you used to find the det. (eg, “swapping rows changes the sign of the det” or “the determinant of the derivative of a matrix is the matrix of its integral” (this second fact is nonsense))

2a)  $M = \begin{bmatrix} 1 & a & r \\ 2 & b & s \\ 4 & c & t \end{bmatrix}$

2b)  $N = \begin{bmatrix} 1 & 2 & 4 \\ a & b & c \\ r+2 & s+4 & t+8 \end{bmatrix}$

2c)  $Q = \begin{bmatrix} 10 & 20 & 40 \\ a & b & c \\ r & s & t \end{bmatrix}$

2d)  $R = \begin{bmatrix} 10 & 20 & 40 \\ a & b & c \\ 1 & 2 & 4 \end{bmatrix}$

2e)  $S = 4B$

3) Suppose the following elementary row operations turn the matrix  $A$  into  $U$ :  
First, rows  $r_1$  and  $r_2$  are swapped. Second,  $r_3 \leftarrow r_3 + 10r_2$ , Third, row 1 is multiplied by 4. The matrix  $U$  is a  $3 \times 3$  upper triangular matrix, with main diagonal entries 4, 3, and 2.

3A) What is  $\det(A)$ ? Explain.

3B) Is  $A$  invertible? Explain how you know.