

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers.

1. Evaluate the following integrals.

(a) $\int \cos^{12} x \sin^3 x \, dx$

(b) $\int x^2 \sin x \, dx$

(c) $\int \frac{4x^2 - 7x + 1}{(x-1)^2(x+1)} dx$

(d) $\int \frac{\sqrt{4-x^2}}{x^2} dx$

2. The following integral is improper. Either compute its value to show it converges, or show it diverges.

$$\int_0^3 \frac{1}{\sqrt[3]{x-1}} dx$$

3. Consider the function $f(x) = \ln(1 + x)$.

(a) Find $P_3(x)$, the 3rd order Taylor Polynomial, of $f(x)$ centered at $x = 0$. Simplify your answer as much as possible, in other words, fractional coefficients must be in lowest terms.

(b) Use $P_3(x)$ to find estimates for $\ln(0.9)$ and $\ln(1.3)$. Show your work.

(c) Use Taylor's Theorem to approximate the error of your estimate for $\ln(0.9)$, from part (b), on the interval $[-0.5, 0.4]$. Recall that error bounds for estimates using a Taylor Polynomial $P_n(x)$ may be determined using:

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!} |x - x_0|^{n+1}.$$

[BONUS] Which of the approximations in part (b) would you expect to be more accurate? Briefly explain. (Note: simply evaluating $\ln(0.9)$ and $\ln(1.3)$ on your calculator to see which is closer to the estimates in (b) will not earn you any bonus points.)

4. Consider the initial value problem $\frac{dy}{dx} = xe^{-y}$ with $y(0) = 3$. Use separation of variables to find the solution to the IVP. Be sure to use the initial condition to determine the values of any constants.

5. Consider $\int_0^{\infty} \frac{1}{e^{2x} + 2700x} dx$. Use a comparison to determine if this integral converges or diverges? If it converges find an upper bound for its value.

6. The magnitude of an earthquake, as measured on the Richter scale, recorded in a region of North America is a random variable X with probability density given by

$$f(x) = \begin{cases} 0.417e^{-0.417x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

An earthquake struck Maine on October 16, 2012 measuring 4.6 on the Richter scale. Find the probability that an earthquake striking North America will exceed a magnitude of 4.6 on the Richter scale.

[BONUS] What is your favorite movie?

Useful Formulas

Trigonometric Identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta = 1 + \tan^2 \theta$
- $\sin(2\theta) = 2 \sin \theta \cos \theta$
- $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$
- $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$

Some antiderivative rules:

- $\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C$
- $\int \csc \theta \, d\theta = \ln |\csc \theta - \cot \theta| + C$
- $\int \csc \theta \cot \theta \, d\theta = -\csc \theta + C$
- $\int \csc^2 \theta \, d\theta = -\cot \theta + C$
- $\int \tan \theta \, d\theta = -\ln |\cos \theta| + C$
- $\int \cot \theta \, d\theta = \ln |\sin \theta| + C$