1. Consider the function $f(x) = x^{(x^3-4x)}$ (You can write $y = x^{(x^3-4x)}$ if you prefer).

1A: Use logarithmic differentiation to find the derivative f' of f. (You can write y' and y if you like). Express the result in terms of x).

let $y = x^{(x^3-4x)}$

so $\ln y = \ln\left(x^{(x^3-4x)}\right) = (x^3-4x)\cdot(\ln x)$ ← (using the product rule on this)

taking derivatives: $\frac{1}{y}y' = (3x^2-4)(\ln x) + (x^3-4x)\frac{1}{x}$

so $y' = \left[(3x^2-4)(\ln x) + (x^3-4x)\frac{1}{x}\right]\cdot x^{(x^3-4x)}$ (we multiplied both sides of the previous eqn by y, which is $x^{(x^3-4x)}$)

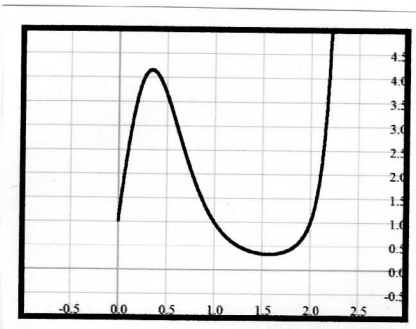
1B: Use the answer to 1A to find the slope of the line tangent to the graph of f at the point $(1, f(1))$. Show all your steps.

we need $y'\big|_{x=1} = \left[(3-4)(\ln 1) + (1-4)\frac{1}{1}\right]\cdot 1^{(1-4)}$

$= \left[-1\cdot 0 + (-3)\right]\cdot 1^{-3}$

$= -3$

1C: Have your calculator draw the graph of f in the window [Xmin, Xmax] × [Ymin, Ymax] = $[-1, 3] \times [-1, 5]$ and make an excellent facsimile of the result in the space to the right:



1D: Using the calculator's "maximum" function (it's on the same menu as the "zero" function) to locate the x-coordinate of the local max you see in 1C. Tell me what you used for your LeftBound, RightBound and your Guess, then also give me the value of x and y at that point (to as many places as your calculator gives you).

LeftBound = [.25] RightBound= [.5] Guess= [.35]

x coord of local max: [.34465727] y coord of local max: [4.1574892]

} there are MANY possible choices for all three answers... mine are representative

NOTE that different choices of LBound & RBound can produce slightly different results!!!

2. Let $f(x) = x^3\sin(5x)$.

2A. Find $f'(x)$ using the product and chain rules in the usual way.

$f'(x) = 3x^2\sin(5x) + x^3(\cos 5x)\,5$

2B. Find $f'(x)$ using logarithmic differentiation, and simplify the result so it's the same as in 2A. *Show all your steps!*

let $y = x^3\sin 5x$

so $\ln y = \ln(x^3\cdot\sin 5x)$

$= \ln x^3 + \ln(\sin(5x))$

$= 3\ln x + \ln(\sin(5x))$

next: $\frac{1}{y}y' = \frac{3}{x} + \frac{1}{\sin 5x}\cdot\cos(5x)\cdot 5$

so $y' = \left[\frac{3}{x} + \frac{\cos(5x)\,5}{\sin 5x}\right]\left[x^3\sin 5x\right]$ → $= 3x^2\sin 5x + (\cos(5x)5)\cdot x^3$